Economic Growth, Fall 2010
Instructor: Dmytro Hryshko
Problem Set 2 (32 points). **Due November, 12.**

1. (**10 points**) Consider a version of the Solow model where the aggregate production function is

$$Y = F(K, L, Z) = K^{\alpha} L^{\beta} Z^{1-\alpha-\beta},$$

where Z is land, available in fixed inelastic supply (that is, constant). Assume that $\alpha + \beta < 1$, capital depreciates at the rate δ, and saving rate is s.

   (a) (**2 points**) Suppose first there is no population growth. Find the steady-state capital per worker, and the steady-state output per worker.

   (b) (**4 points**) Now suppose there is population growth at the rate n, $\frac{\dot{L}}{L} = n$. What happens to the capital per worker and output per worker as time goes to infinity (in the very long run)? Show all the necessary work needed to arrive at your conclusion. (*Hint*: redefine the production function so that $Y = K^{\alpha} \tilde{L}^{1-\alpha}$, where $\tilde{L}^{1-\alpha} = L^{\beta} Z^{1-\alpha-\beta}$ and so $\tilde{L} = L^{\frac{\beta}{1-\alpha}} Z^{\frac{1-\alpha-\beta}{1-\alpha}}$; then you can find the steady state for $K/\tilde{L}$ and $Y/\tilde{L}$; finally, you can see the growth-rate of K/L in the very long run since $\frac{K}{L} = (K/\tilde{L})(\tilde{L}/L)$.)

   (c) (**4 points**) What happens to the wage rate and the price of land (assume that land is paid its marginal product; that is, the markets for all inputs are perfectly competitive) as time goes to infinity (in the very long run)? Show all the necessary work needed to arrive at your conclusion.

2. (**8 points**) Assume everything needed to arrive at the Figure 5.1. The economy is currently on its balanced-growth path. Assume now that aliens took away a sizeable portion of researchers but everything else remains unchanged.

   (a) (**2 points**) Draw a diagram similar to Figure 5.1.

(b) (**2 points**) Draw a diagram similar to Figure 5.3.

(c) (**2 points**) Sketch a path of $\log\left[\frac{Y(t)}{L(t)}\right]$ against time.

(d) (**2 points**) Discuss briefly how the economy evolves at the time and after the shock.

3. (**10 points**) Chapter 6, problem 5. (each sub-question 2 points)

4. (**4 points**, each sub-question 2 points) Chapter 7, problem 2, (a) and (b). (*Hint*: take 2 countries, poor (p) and rich (r) so that $I_r/I_p = 10$. Now you want to explain this difference by postulating a general production function $Y = (u_K K)^\alpha (u_H h L)^{1-\alpha}$, where u_K and u_H are utilization rates of physical and human capital, respectively; you would need to simplify the function for answering (a) and (b).)