

Economic Growth, Fall 2010
Instructor: Dmytro Hryshko
Problem Set 1 (24 points). Due October, 15.

1. **(3 points)** Suppose that the growth rate of some variable, X , is constant and equal to $a > 0$ from time 0 to time t_1 ; drops to $0 < a_1 < a$ at time t_1 ; rises gradually from a_1 to a from time t_1 to time t_2 ; and is constant and equal to a after time t_2 .
 - (a) **(1 points)** Sketch a graph of the growth rate of X as a function of time.
 - (b) **(2 points)** Sketch a graph of natural log of $X(t)$, $\log X(t)$, as a function of time.
2. **(5 points)** Consider a Solow model without technological progress, in its steady state. Now suppose that the rate of population growth falls.
 - (a) **(3 points)** What happens to the steady-state values of capital per worker, output per worker, and consumption per worker? Sketch the paths of these variables as the economy moves to its new steady state.
 - (b) **(2 points)** Sketch a graph of $\log Y(t)$ and $\log \frac{Y(t)}{L(t)}$ against time.
3. **(10 points)** Assume the “Solow” economy is currently in the steady state. The savings rate $s = 20\%$; production function is $Y = K^{4/10}L^{6/10}$; population growth rate, n and depreciation rate, δ are some positive constants. (You don’t need to know what they are to answer the questions, but you may pick any numbers if it helps.) The government decides to tax both wage and capital income at the proportional rate $0 < \tau < 1$. Thus, consumers receive real income in the amount equal to $(1 - \tau)Y$. Assume that the government invests the full amount of the tax proceedings. Thus, government savings are τY .
 - (a) **(5 points)** Would you observe any changes in the economy resulting from the tax policy and, if yes, what changes (steady-state output per worker? steady-state capital per worker?)? Show a graph.
 - (b) **(5 points)** If the governments wants to set τ to move the economy towards the golden-rule steady-state, what value of τ would it choose?

4. (6 points) Consider the Solow growth model with constant savings rate s and depreciation rate of capital equal to δ . Assume that population is constant and the aggregate production function is given by the constant returns to scale production function

$$F(A_K(t)K(t), A_L(t)L(t)),$$

where $\dot{A}_L(t)/A_L(t) = g_L > 0$ and $\dot{A}_K(t)/A_K(t) = g_K > 0$. Suppose that F is Cobb-Douglas. Determine the balanced-path growth rate in output per worker, the steady-state value of capital per effective worker, and the law of motion of capital per effective worker. (Assume that production function is of the form $Y = (A_K K)^\alpha (A_L L)^{1-\alpha}$). The trick is to define the labor-augmenting technological progress by choosing $A(t)$ which will be a mix of $A_K(t)$ and $A_L(t)$ as we've done in class, and then proceed as usual.)