Economic Growth, Fall 2009 Instructor: Dmytro Hryshko

Midterm Exam (35 points). Friday, October 30.

General Instructions: You should answer questions 1, 2, and 3 and one question of your choice, either 4 or 5.

1. (10 points) Assume the "Solow" economy is currently in the steady state; for simplicity, you may assume that g = 0. The government decides to tax both wage and capital income at the rate τ . Thus, consumers receive real income in the amount equal to $(1 - \tau)Y$. Assume that the government invests a fraction s of the tax proceedings. Thus, government savings are $s\tau Y$ (assume that the rest of the tax proceedings $\tau(1-s)Y$ do not have any use in the economy). Would you observe any changes in the economy resulting from the tax policy and, if yes, what changes (steady-state output per worker? steady-state consumption per worker? steady-state capital per worker?)? Show a graph.

2. (10 points) Consider the world economy that consists of three sectors: the final-goods sector, the intermediate capital goods sector, and the research sector. In the final goods sector, aggregate production function is $Y = K^{\alpha}(AL_Y)^{1-\alpha}$. Labor force, L, is comprised of workers engaged in producing ideas, L_A , and workers, engaged in the final-goods sector, L_Y : $L = L_Y + L_A$. Also, $L_A = s_R L$, where $0 < s_R < 1$. Total working population grows at the rate n, capital depreciates at the rate δ , consumers save a constant fraction of income s, and the stock of technology grows at the rate g_A in the steady state. Production function of new ideas is $\dot{A} = dL_A$. Derive an equation for output per worker, $\left(\frac{Y}{L}\right)(t)$, on a balanced growth path in this world economy.

3. (10 points) Continue with the set-up of the previous problem. Assume the economy is on a balanced-growth path. Suppose there is a permanent *decline* in the share of the working population engaged in research. What happens to the growth rate and the level of technology over time? Draw appropriate graphs.

4. (5 points) Consider two initially closed "Solow" economies that are the same in every respect but one is in the steady state and another is poorer in terms of capital per worker and output per worker. For simplicity, assume that technological growth in both economies is zero. Now assume that the economies open their borders so that both capital and labor can flow from one economy to another. Where would capital flow, from the poor to the rich economy, or vice versa, and why? Where would labor flow, from the poor to the rich economy, or vice versa, and why?

5. (5 points) Let the production function be Cobb-Douglas, of labor-augmenting type: $Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$. Show that we can rewrite this production function so that technological progress is of capital-augmenting type $Y(t) = (B(t)K(t))^{\alpha}L(t)^{1-\alpha}$, or technological progress is Hicks-neutral: $Y(t) = M(t)K(t)^{\alpha}L(t)^{1-\alpha}$. That is, express B(t) and M(t) as functions of A(t) and α .