One-Sector Models of Endogenous Growth

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1. Mid-1980s: dissatisfaction with exogenously driven explanations of long-run productivity growth.

2. It led to construction of models in which the key determinants of growth were endogenous to the model. Hence, the reason for the name endogenous growth.

The AK Model

The key property of endogenous-growth models is the absence of diminishing returns to capital.

The simplest version of a production function with this property (more realistic if we include human capital into K)

$$Y = AK,$$

where $A > 0$ is a constant that reflects the level of technology.

Output per capita is $y = Ak$ and the average and marginal products of capital are constant at A.

$$g_k = \frac{\dot{k}}{k} = sA - (n + \delta).$$

Positive long-run growth in k, y, and c is possible if $sA > n + \delta$, without the need for any technological progress.

However, the model does not predict absolute or conditional convergence since the growth rate in per-capita income does not depend on the level of per-capital income.

Endogenous growth with transitional dynamics

$$g_k = s\frac{f(k)}{k} - (n+\delta).$$

Intuition: need to have a declining average product of capital that converges to a constant sA in the very long run so that g_k is high for low levels of k but approaches to a constant $sA - (n+\delta)$ in the long run.

Marginal product of capital should converge to a constant in the long-run—violation of one of the Inada conditions.

Endogenous growth with transitional dynamics

$$Y = AK + BK^{\alpha}L^{1-\alpha}$$
$$y = Ak + Bk^{\alpha}$$

$$g_k = \frac{\dot{k}}{k} = s\left[A + Bk^{-(1-\alpha)}\right] - (n+\delta).$$

As k gets large, g_k tends to $sA - (n+\delta)$ as in the AK model. If two economies differ only in terms of their initial values $k(0)$, the one with the smaller capital stock per person will grow faster in per capita terms—conditional convergence.

Other models

Experience with production or investment contributes to productivity.

A larger economy-wide stock of capital improves the level of the technology for each producer.

Diminishing returns to capital may not apply in the aggregate, and increasing returns possible (in the latter case, $f(k)/k$ increases with k and g_k is increasing over some range of k).

A one-sector model with physical and human capital

AK model: K viewed broadly to include human and physical components.

Goal: make this interpretation explicit.

$$Y = F(K, H),$$

F is CRS in K and H, human capital. Since $\lambda Y = F(\lambda K, \lambda H)$, we can define $\lambda = 1/K$ and

$$Y = K \times f(H/K),$$

where $f(H/K) = F(1, H/K)$ and $f'(\cdot) > 0$.

Assumptions

- Output can be used for consumption, investment in K, or investment in H.
- H and K depreciate at the rates δ_H and δ_K, respectively.
- L is constant.
- Factor markets competitive; R_K and R_H are the rental prices of the two types of capital.

Since $Y = Kf(H/K)$,

$$\begin{aligned} F_K =& f(H/K) + Kf'(H/K)(-H/K^2) = \\ & f(H/K) - (H/K)f'(H/K) = R_K. \\ F_H =& Kf'(H/K)(1/K) = f'(H/K) = R_H. \end{aligned}$$

The two types of capital should attract the same rates of return:

$$R_K - \delta_K = R_H - \delta_H.$$

This implies

$$\begin{aligned} f(H/K) - (H/K)f'(H/K) - \delta_K = f'(H/K) - \delta_H \\ f(H/K) - f'(H/K)[1 + H/K] = \delta_K - \delta_H. \end{aligned}$$

$$f(H/K) - f'(H/K)[1 + H/K] = \delta_K - \delta_H.$$

Thus, there's a unique steady-state value of H/K.

If we define $A = f(H/K)$,

$$Y = Kf(H/K) = AK.$$

If we regard CRS to the two kinds of capital as plausible, the AK model is a valid representation of the broader model.

Models with learning-by-doing and knowledge spillovers

- The key to endogenous growth in the AK model is the absence of diminishing returns to the factors than can be accumulated.

- Romer (1986): eliminated the tendency for diminishing returns by assuming that knowledge creation is a side product of investment—a firm that increases its physical capital learns simultaneously how to produce more efficiently.

- The positive effect of experience on productivity is called learning-by-doing.

Firm i's technology:

$$Y_i = F(K_i, A_i L_i),$$

where A_i is the index of technology available to the firm.

- Regular assumptions about production function.
- Don't assume that A_i grows exogenously.
- An increase in a firm's capital stock increases the stock of knowledge, A_i (motivation: large positive effects of experience on productivity in shipbuilding; patents follow investment in physical capital).
- Assume the aggregate labor force, L, is constant.

A_i is a public good—a piece of knowledge spills over across the whole economy.

$\dot{A}_i$ is proportional to K. Discoveries are unintended by-products of investment. Production function becomes

$$Y_i = F(K_i, KL_i).$$

Diminishing returns to K_i, holding K and L_i constant but constant returns to K_i and K combined—this will yield endogenous growth.

A Cobb-Douglas example

$$Y_i = AK_i^{\alpha}(KL_i)^{1-\alpha}, \;\; 0 < \alpha < 1.$$

Define $y_i = \frac{Y_i}{L_i}$, $k_i = \frac{K_i}{L_i}$. In equilibrium, $k_i = k = \frac{K}{L}$ and $y_i = y$. Thus,

$$\frac{y}{k} = \frac{Ak^{\alpha}K^{1-\alpha}}{k} = \frac{Ak^{\alpha}k^{1-\alpha}L^{1-\alpha}}{k} = AL^{1-\alpha}.$$

y/k is invariant to k and increasing in L.

Scale effects

The model implies a scale effect—an expansion of the aggregate labor force, L, raises per capita growth rate. If the labor force grows over time, the per capita growth rates would increase over time.

- If L is identified with the aggregate labor force, the prediction is that countries with more workers tend to grow faster in per capita terms (weak positive relation in the data).
- Is the relevant scale variable world population (Kremer 1993), or close neighbors (e.g., industries within a country)?