Chapter 9. Natural Resources and Economic Growth

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We want to understand growth in the presence of the earth's finite supply of arable land and nonrenewable natural resources (e.g., oil and natural gas).

In the Solow model, the presence of (depletable) natural resources *reduces* the long-run growth rate of the economy. Perhaps, the size of this reduction was not that big, at 0.3%.

Let T be the (fixed) amount of land available for production in each period. Aggregate production function is:

$$Y = BK^{\alpha}T^{\beta}L^{1-\alpha-\beta},$$

where $0 < \beta < 1$ and $\alpha + \beta < 1$. *B* is an index of technological progress. Production function is constant returns to scale in *K*, *L*, and *T* (replication argument). Furthermore,

$$\begin{split} \frac{\dot{B}}{B} &= g_B, \\ \frac{\dot{L}}{L} &= n, \\ \dot{K} &= sY - \delta K. \end{split}$$

Along a balanced growth path, the growth rates of K/L and Y/L are constant and equal. Thus, $\frac{K/L}{Y/L} = \frac{K}{Y}$ should be constant.

Divide production function through by Y^{α} , to obtain

$$Y^{1-\alpha} = B\left(\frac{K}{Y}\right)^{\alpha} T^{\beta} L^{1-\alpha-\beta},$$

and so

$$Y = B^{\frac{1}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} T^{\frac{\beta}{1-\alpha}} L^{\frac{1-\alpha-\beta}{1-\alpha}}.$$

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Since we assumed that T is constant, along the balanced growth path, when K/Y is constant, total output grows at the rate $g_Y = \frac{g_B}{1-\alpha} + (1 - \frac{\beta}{1-\alpha})n.$

The growth rate of output per worker, therefore, is $g_y = g_Y - n = \frac{g_B}{1-\alpha} - \frac{\beta}{1-\alpha}n = g - \bar{\beta}n$, where $g \equiv \frac{g_B}{1-\alpha}$ and $\bar{\beta} \equiv \frac{\beta}{1-\alpha}$.

- The long-run growth rate of the economy now depends on population growth, n, and the importance of land in production, β. There is a "race" between technological progress and the diminishing returns due to the fixed amount of land.
- There are decreasing returns to K and L in the presence of a fixed supply of land. Absent technological progress, when g = 0, the growth rate of output per worker is *negative*, and output per worker will approach zero in the very long run.
- The growth rate in B may potentially offset the pressure of population on the fixed resource and lead to sustained growth in output per worker. The more important is land in production (the higher is β), the lower the long-run growth will be: in this case, the diminishing returns to capital and labor are stronger.

Land was in fixed supply but not subject to depletion.

Introduce natural resources used in production that can be depleted (e.g., natural gas, coal, oil).

Suppose the aggregate production function is constant returns to scale in E, L and K:

$$Y = BK^{\alpha}E^{\gamma}L^{1-\alpha-\gamma},$$

where E is the amount of energy used in production, and $\alpha + \gamma < 1$.

Let the initial amount of resource be R(0).

The stock of resource is depleted as: $\dot{R} = -E$.

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Assume that a constant fraction of the remaining stock of resource is used for energy production each period:

$$E = s_E R, \ 0 < s_E < 1.$$

Thus, $\frac{\dot{R}}{R} = -\frac{E}{R} = -s_E$, and $R(t) = R(0) \exp(-s_E t).$
Since $E(t) = s_E R(t), \ E(t) = s_E R(0) \exp(-s_E t)$ —the stock of remaining resources and the amount of energy used in production decline over time at the rate s_E .



FIGURE 9.1 THE ENERGY STOCK R OVER TIME

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We can express the production function as

$$Y = B^{\frac{1}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} E^{\frac{\gamma}{1-\alpha}} L^{1-\frac{\gamma}{1-\alpha}},$$

 or

$$Y = B^{\frac{1}{1-\alpha}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \left[s_E R(0) \exp(-s_E t)\right]^{\frac{\gamma}{1-\alpha}} L^{1-\frac{\gamma}{1-\alpha}}.$$

Along the balanced growth path, Y grows at the rate

$$g_Y = \frac{g_B}{1-\alpha} - s_E \frac{\gamma}{1-\alpha} + n\left(1 - \frac{\gamma}{1-\alpha}\right).$$

Thus, the growth rate of output per worker, along the balanced growth path, is

$$g_y = g_Y - n = \frac{g_B}{1 - \alpha} - \frac{\gamma}{1 - \alpha} (s_E + n) = g - \bar{\gamma} (s_E + n),$$

where $g = \frac{g_B}{1-\alpha}$ and $\bar{\gamma} = \frac{\gamma}{1-\alpha}$.

- Higher population growth leads to increased pressure on the finite resource stock and reduces the growth in output per worker.
- An increase in the depletion rate, s_E , reduces the long-run growth rate of the economy.

The accumulation of capital and labor runs into diminishing returns since land and nonrenewable resources are in limited supply. For a model with both fixed land and nonrenewable resources, the growth rate in output per capita is:

$$g_y = g - \underbrace{\left[(\bar{\beta} + \bar{\gamma})n + \bar{\gamma}s_E \right]}_{\text{"growth drag"}}.$$

Nordhaus (1992): $\beta = 0.1$, $\gamma = 0.1$, $\alpha = 0.2$, n = 0.01, and $s_E = 0.005$, where β , for example, is the land's share of output (payments to land as a share of GDP). Thus, the growth "drag" is estimated at about 0.0031—annual per capita growth of output in the U.S. is about 0.31% lower due to the presence of a fixed supply of land and depletable resources.

Is the annual "loss" of 0.31% large or small? A quantity, growing at 0.31%, will double in about 225 years.

Consider the following question: If you start with income y_0 , how much would you be willing to pay annually to have your income growing at 2.1% instead of 1.8%? For an interest rate r = 0.06, you will be willing to pay at most 7.1% of annual income.

We want to solve for κ in the following equation:

$$(1-\kappa)y_0\Sigma_1=y_0\Sigma_2,$$

where
$$\Sigma_1 = 1 + \frac{1+0.021}{1+0.06} + \left(\frac{1+0.021}{1+0.06}\right)^2 + \dots$$
, and
 $\Sigma_2 = 1 + \frac{1+0.018}{1+0.06} + \left(\frac{1+0.018}{1+0.06}\right)^2 + \dots$

$$\Sigma_1 = \frac{1}{1 - \frac{1 + 0.021}{1 + 0.06}} = \frac{1.06}{0.06 - 0.021}$$

$$\Sigma_1 = \frac{1}{1 - \frac{1 + 0.018}{1 + 0.06}} = \frac{1.06}{0.06 - 0.018}$$

Suppose the production function is $Y = K^{\alpha}T^{\beta}E^{\gamma}L^{1-\alpha-\beta-\gamma}$.

In competitive factor markets, each factor is paid its marginal product.

For example,
$$R = F_K = \alpha \frac{Y}{K}$$
. The share of output paid to capital is $v_K = \frac{RK}{Y} = \alpha$. Similarly, $v_T = \beta$, $v_E = \gamma$, $v_L = 1 - \alpha - \beta - \gamma$.

The Cobb-Douglas function implies that all shares are constant over time. However, in the data, v_T and v_E are falling over time. Note that a factor scarce in supply but high in demand will have a high price.

$$\frac{v_E}{v_L} = \frac{P_E E/Y}{wL/Y} = \frac{P_E E}{wL}$$
$$\frac{P_E}{w} = \frac{v_E/v_L}{E/L}.$$

As L grows and E gets depleted, E becomes relatively scarce and, for constant income shares v_E and v_L , the price of nonrenewable resources should rise relative to the price of labor. The same applies to the relative price of land, P_T/w . In the U.S. data for fossil fuels (oil, natural gas and coal),

- P_E/w is falling, perhaps, because
- v_E is falling, and
- E/L is rising (maybe, world continues to discover new deposits of fossil fuels).







To explain the declining resource share in output, we will use the Constant Elasticity of Substitution (CES) production function. With only two factors of production, capital and energy, the production function is:

$$Y = F(K, E) = (K^{\rho} + (BE)^{\rho})^{1/\rho},$$

where B is an index of technology.

Elasticity of substitution is defined as

 $\frac{\Delta \left(\frac{K}{E} \right) / \left(\frac{K}{E} \right)}{\Delta TRS/TRS},$

where TRS is the technical rate of substitution (the slope of an isoquant).

Define $K^{\rho} + (BE)^{\rho} \equiv z$ and note that $\Delta Y \approx F_K \Delta K + F_E \Delta E$. Thus,

$$\Delta Y = z^{1/\rho - 1} K^{\rho - 1} \Delta K + z^{1/\rho - 1} B^{\rho} E^{\rho - 1} \Delta E.$$

Along the isoquant, $\Delta Y = 0$, and therefore $TRS = \frac{\Delta K}{\Delta E} = -B^{\rho} \left(\frac{K}{E}\right)^{1-\rho}$.

The percentage change in TRS can be calculated from $d\ln|TRS|$, and the percentage change in $\left(\frac{K}{E}\right)$ as $d\ln\left(\frac{K}{E}\right)$.

$$\begin{aligned} \ln |TRS| &= \rho B^{\rho-1} + (1-\rho) \ln \frac{K}{E}, \text{ or} \\ \ln \left(\frac{K}{E}\right) &= \frac{1}{1-\rho} \ln |TRS| - \frac{\rho}{1-\rho} B^{\rho-1}. \end{aligned}$$

Since $\frac{\Delta(\frac{K}{E})/(\frac{K}{E})}{\Delta TRS/TRS} &= \frac{d \ln(\frac{K}{E})}{d \ln |TRS|}, \text{ the elasticity of substitution, } \sigma, \text{ is} \\ \text{equal to } \frac{1}{1-\rho}. \end{aligned}$

Note that if $\rho = 0$, $|TRS| = \frac{K}{E}$, and the elasticity of substitution is equal to 1.

The elasticity of substitution is greater than 1 if $0 < \rho < 1$ and less than 1 if $\rho < 0$. Higher values of ρ imply greater substitutability between the factors of production: e.g., when $\rho = 1, \sigma = \infty$ —production function is linear in K and E.

Energy's share

The share of energy in output, provided the markets are competitive, is $v_E = \frac{F_E E}{Y}$.

$$F_E = \frac{1}{\rho} \rho B^{\rho} E^{\rho-1} \left(K^{\rho} + (BE)^{\rho} \right)^{\frac{1}{\rho}-1}.$$

And so

$$v_E = \frac{F_E E}{Y} = \frac{B^{\rho} E^{\rho} (K^{\rho} + (BE)^{\rho})^{\frac{1}{\rho} - 1}}{(K^{\rho} + (BE)^{\rho})^{\frac{1}{\rho}}} = \frac{B^{\rho} E^{\rho}}{\left[(K^{\rho} + (BE)^{\rho})^{\frac{1}{\rho}}\right]^{\rho}} = \left(\frac{BE}{Y}\right)^{\rho}.$$

Empirically, $\frac{E}{Y}$ is falling over time. If $\rho > 0$ and B is not growing rapidly, the share of energy will decline over time:

$$\frac{\dot{v}_E}{v_E} = \rho \underbrace{\frac{\dot{B}}{B}}_{+} + \rho \underbrace{\frac{\left(\frac{\dot{E}}{Y}\right)}{E/Y}}_{-}.$$

Intuition

- If $\rho > 0$, energy and capital become more substitutable; when energy becomes relatively more expensive, more capital is used to produce the same amount of output; the output share of a more plentiful factor will increase; and energy is not a necessary input into production. However, it might be hard to use capital instead of energy, and energy must be a necessary input.
- If $\rho < 0$, factors are less substitutable; energy is a necessary input; the share of the relatively scarce factor should increase but...the things turn around if energy-specific technological change, B, changes energy from an "increasingly scarce factor" to an "increasingly plentiful factor."

In the U.S. data, the ratio $\left(\frac{E}{Y}\right)$ declined by a factor of 2 between 1949 and 1999, implying that an annual growth rate of $\left(\frac{E}{Y}\right)$ was about -1.4%.

(You can solve for x from $\left(\frac{E}{Y}\right)_{1949} (1+x)^{50} = \frac{1}{2} \left(\frac{E}{Y}\right)_{1949}$.)

If $\rho < 0$, for v_E to be falling over time, the energy-specific technological change should have been increasing at a rate higher than 1.4%.

It sounds quite plausible, taking into account that, in the presence of depletable resources, energy-saving research would be particularly profitable.