

Name: \_\_\_\_\_

**ECON403: Economic Growth, Fall 2008**

**Instructor: Dmytro Hryshko**

**Final Exam (40 points). December 11.**

1. **(5 points)** What are the *proximate* and *fundamental* causes of economic growth? List the four main hypotheses of fundamental causes of economic growth. Pick one of the hypotheses and try to explain why it fails or does not fail in light of some historical data we see.

2. **(14 points)** Consider a model of growth and development. A developing economy uses all of its labor resources in the final goods sector, and utilizes up to  $h$  intermediate capital goods, where the range of intermediate capital goods in the world is  $1, \dots, A$  and  $h \leq A$ . The economy's production function is:  $Y = L^{1-\alpha} \sum_{j=1}^h x_j^\alpha$ . Production function for adoption of new ideas in the economy relates to the stock of ideas in the world ("technological frontier"),  $A(t)$ , the stock of already adopted ideas in the economy,  $h(t)$ , trade openness,  $\mu$ , and the average schooling in the economy,  $u$ :  $\dot{h}(t) = \mu \exp(\psi u) A(t)^\gamma h(t)^{1-\gamma}$ .  $\psi$  is a constant,  $0 < \gamma < 1$ . Assume that  $A$  grows exponentially at a constant rate  $g$ .

(a) **(2 points)** Assuming that  $x_j = x$  for all  $j$ , and the sum of  $x_j$ 's at time  $t$  should equal to the supply of capital in equilibrium, express aggregate production function in terms of aggregate labor,  $L(t)$ , and aggregate capital,  $K(t)$ .

(b) **(2 points)** Assuming that on a balanced growth path  $h$  and  $A$  grow at the same rate, find the steady-state ratio of  $A/h$ .

(c) **(2 points)** Find output per worker on a balanced growth path.

(d) (**2 points**) Construct a graph with  $\dot{h}/h$  on the vertical axis and  $A/h$  on the horizontal axis assuming that  $\gamma = 1$ .  $A/h$  in equilibrium will be pinned down by the intersection of the  $\dot{h}/h$  line as a function of  $\mu$ ,  $\psi$ ,  $u$ , and  $A/h$ , and the equilibrium line  $\dot{h}/h = g$ .

(e) (**2 points**) Assume that the economy's average schooling  $u$  increases to  $u'$  so that  $u' > u$ . Assume that before this change the economy was on its balanced growth path. Plot the behavior of  $h/A$  over time.

(f) (**2 points**) Plot the behavior of  $\log(h(t))$  over time.

(g) (**2 points**) Discuss the effects of the change in  $u$  on the long-run growth and level of output per capita.

3. (**6 points**) Let the aggregate production function be  $Y = AK$ .  $A$  is the state of technology that is constant over time; the savings rate in the economy is  $s$ , depreciation rate is  $\delta$ , and population grows exponentially at a constant rate  $n$ .

(a) (**2 points**) Find the growth rate of output per capita as a function of the economy's fundamentals,  $s$ ,  $A$ ,  $n$ , and  $\delta$ .

(b) (**2 points**) Do policies that affect, for example,  $s$  have permanent growth and/or level

effects?

(c) (**2 points**) Provide some arguments why the  $AK$  model is not appropriate model for describing cross-country (or within-country) growth experiences.

4. (**5 points**) Consider the Lucas model. Aggregate production function is  $Y = K^\alpha(hL)^{1-\alpha}$ , where  $h$  is human capital per person. Human capital evolves as:  $\dot{h} = (1 - u)h$ , where  $u$  is time spent working and  $(1 - u)$ —time spent accumulating human capital. What is the growth rate of output per capita on a balanced growth path? What are the effects of policies that reduce the economy's population growth rate,  $n$ ?

5. (**10 points**) Consider the following aggregate production function  $Y = F(K, E) = (K^\rho + (BE)^\rho)^{1/\rho}$ , where  $K$  is aggregate capital,  $E$  is the amount of energy used (generated from a non-renewable resource), and  $B$  is an index of technology. The elasticity of substitution between capital and energy is  $\frac{1}{1-\rho}$ .
- (a) (**2 points**) For this production function, if  $\rho$  goes to one is energy absolutely necessary for producing output? What if  $\rho < 0$ ?
- (b) (**4 points**) In the data, we see that the share of aggregate output paid to cover the energy costs,  $v_E$ , is falling over time. Assume that markets for energy, capital and labor are perfectly competitive. Find the share of output paid to energy as a function of  $B$ ,  $E$ ,  $Y$  and  $\rho$ .
- (c) (**4 points**) Find the growth rate of  $v_E$ . Empirically, we observe that  $E/Y$  is falling over time. If  $\rho < 0$  and  $B$  rises over time, would you observe that  $v_E$  is falling over time and why?