Chapter 5. The Engine of Growth

Instructor: Dmytro Hryshko

Endogenous growth theory. The Romer model

- Where does the technological progress come from?
- Is there limit to economic growth and technological progress?
- The theory focuses on understanding the economic forces underlying technological progress (the possibility to earn a profit is important).
- Better thought of as a model for developed economies, where technological progress is driven by R&D in advanced economies.

The basic elements of the model

The final good is produced as:

$$Y = F(K, AL_Y) = K^{\alpha} (AL_Y)^{1-\alpha}, \;\; 0 < \alpha < 1,$$

where L_Y is the amount of labor used to produce the final good Y. $F(K, AL_Y)$ is increasing returns to scale in A, K and L_Y: $F(\lambda K, \lambda A \lambda L_Y) = \lambda^{\alpha+1-\alpha+1-\alpha} K^{\alpha} (AL_Y)^{1-\alpha} = \lambda^{2-\alpha} Y > \lambda Y$.

As before, $\dot{K} = sY - \delta K$, and $\frac{\dot{L}}{L} = n$.

Want to model $\dot{A}$—creation of new ideas (the flow of ideas).

Production function of new ideas

Intuitively, $\dot{A}$ should depend on:

- the amount of labor devoted to creation of ideas, L_A: e.g., $\dot{A} = \bar{d}L_A$—each unit of labor involved in R&D produces $\bar{d}$ new ideas; maybe, more people searching for ideas will tend to duplicate ideas: $\dot{A} = dL_A^{\lambda}$, $\lambda < 1$;
- the available stock of ideas, A: e.g., larger stock, harder to produce ideas $\dot{A} = A^{\phi}L_A$, $\phi < 0$; larger stock, easier to produce ideas $\dot{A} = A^{\phi}L_A$, $\phi > 0$.
  Summing up,

$$\dot{A} = dL_A^{\lambda}A^{\phi}.$$

Allocation of labor $L_A + L_Y = L$, where $L_A = s_R L$, and $L_Y = (1 - s_R)L$.

Growth in the Romer model

In the model, $g_A = g_y = g_k$, where g_A, g_y and g_k are growth rates of ideas, output per worker and capital per worker, respectively.

Use $\dot{A} = dL_A^\lambda A^\phi$, to obtain $\frac{\dot{A}}{A} = dL_A^\lambda A^{\phi-1} = d\frac{L_A^\lambda}{A^{1-\phi}}$. For $\frac{\dot{A}}{A}$ to be constant, L_A^λ should grow at the same rate as $A^{1-\phi}$. Thus, $\lambda\frac{\dot{L}_A}{L_A} = (1-\phi)\frac{\dot{A}}{A} = (1-\phi)g_A$. Note that $\frac{\dot{L}_A}{L_A} = \frac{\dot{L}}{L} = n$, otherwise, labor engaged in research will either exceed the total population (if $\frac{\dot{L}_A}{L_A} > \frac{\dot{L}}{L}$), or cease to exist (if $\frac{\dot{L}_A}{L_A} < \frac{\dot{L}}{L}$).

Thus, $\lambda n = (1-\phi)g_A$, and $g_A = \frac{\lambda n}{1-\phi}$.

Suppose $\lambda = 1$, $\phi = 0$, so that $\dot{A} = dL_A$. Exponential growth is possible only if population grows (more population means more researchers, which means more ideas, which means sustained economic growth).

Some notes

Romer (1990) assumes $\lambda = 1$ and $\phi = 1$: $\dot{A} = dL_A A$, or $\frac{\dot{A}}{A} = dL_A$. This gives a prediction that runs against the data: g_A and the growth rate in output per worker should have been accelerating after 1960 since L_A was accelerating during that period. Thus, ϕ should be less than one.

Even if we model evolution of A, the long-run growth cannot be manipulated by policymakers, for example, by subsidizing R&D.

Growth effects versus level effects

An increase in the share of population doing research (e.g., a subsidy for R&D) to $s'_R > s_R$.

1. Analyze the effects on technological growth and the stock of ideas.
2. Follow the steps of transitional dynamics in the Solow model.

Assume $\phi = 0$ and $\lambda = 1$: $\dot{A} = dL_A$, and $\frac{\dot{A}}{A} = d\frac{L_A}{A} = d\frac{s_R L}{A}$.
Since $\frac{\dot{A}}{A} = g_A$ in steady state, $\frac{L_A}{A} = \frac{g_A}{d}$.

The *level* of technological progress increases permanently, the growth rate reverts to its previous level, g_A.

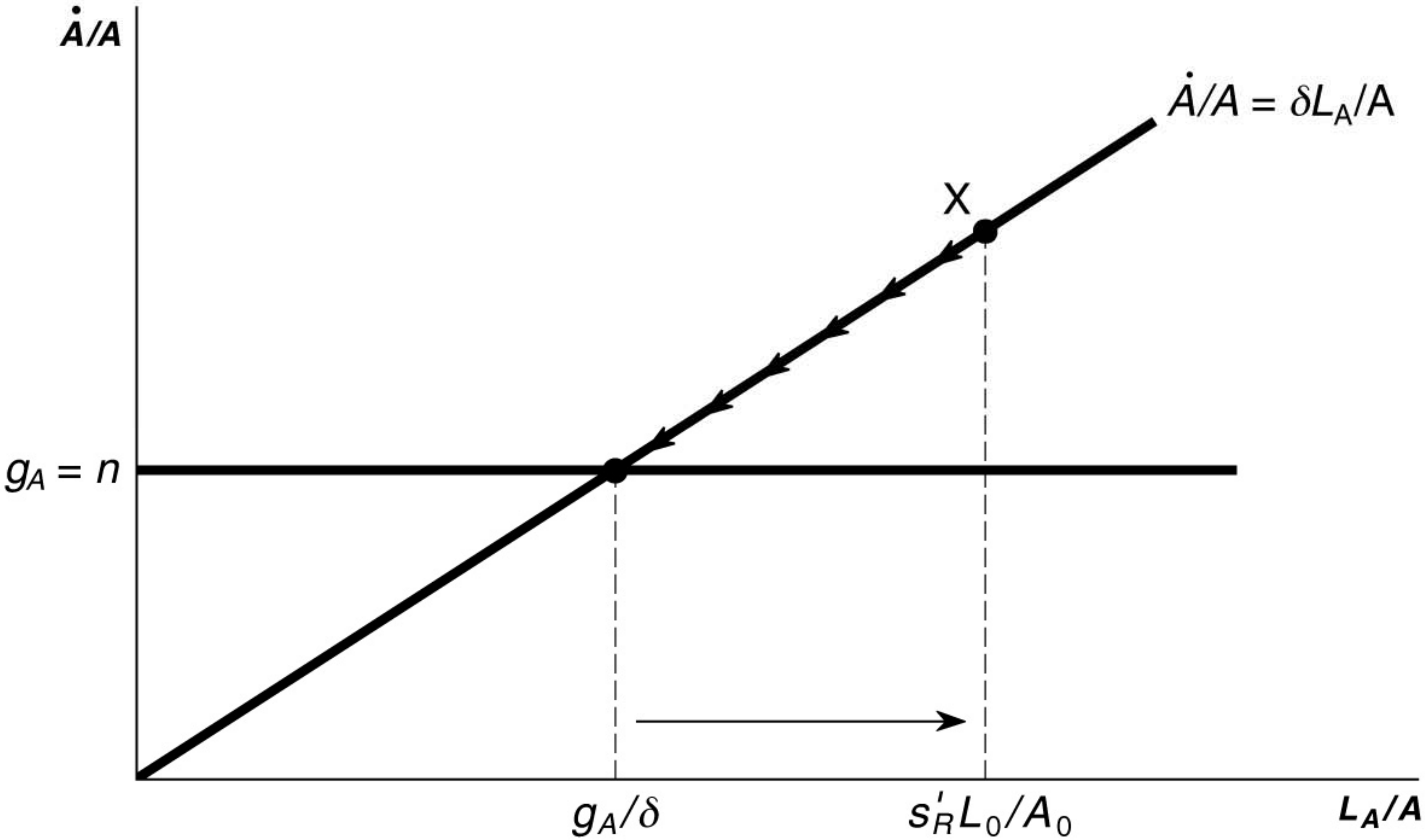

FIGURE 5.1 TECHNOLOGICAL PROGRESS: AN INCREASE IN THE R&D SHARE

Economic Growth, 2nd Edition

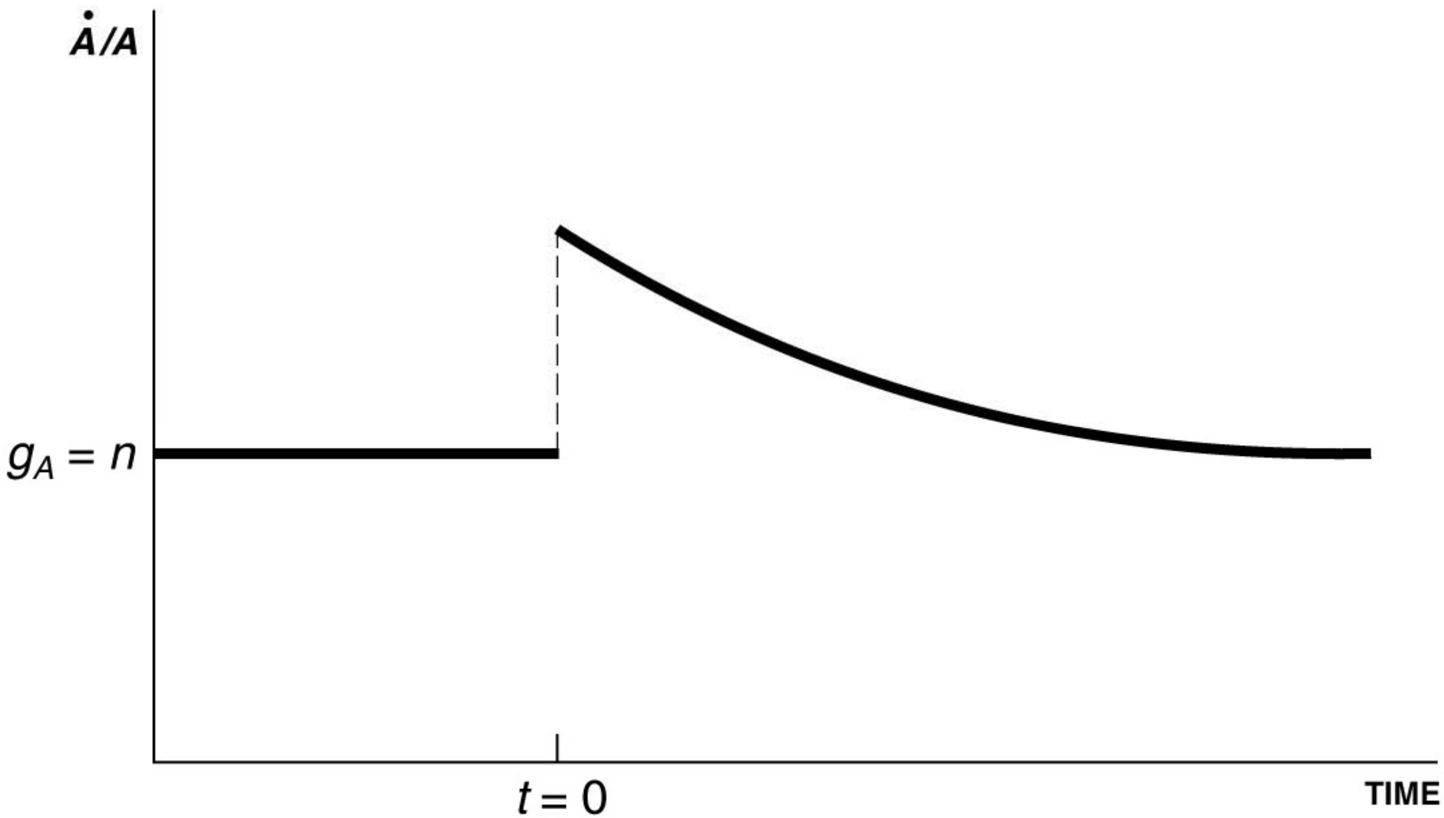

FIGURE 5.2 $\dot{A}/A$ OVER TIME

Economic Growth, 2nd Edition

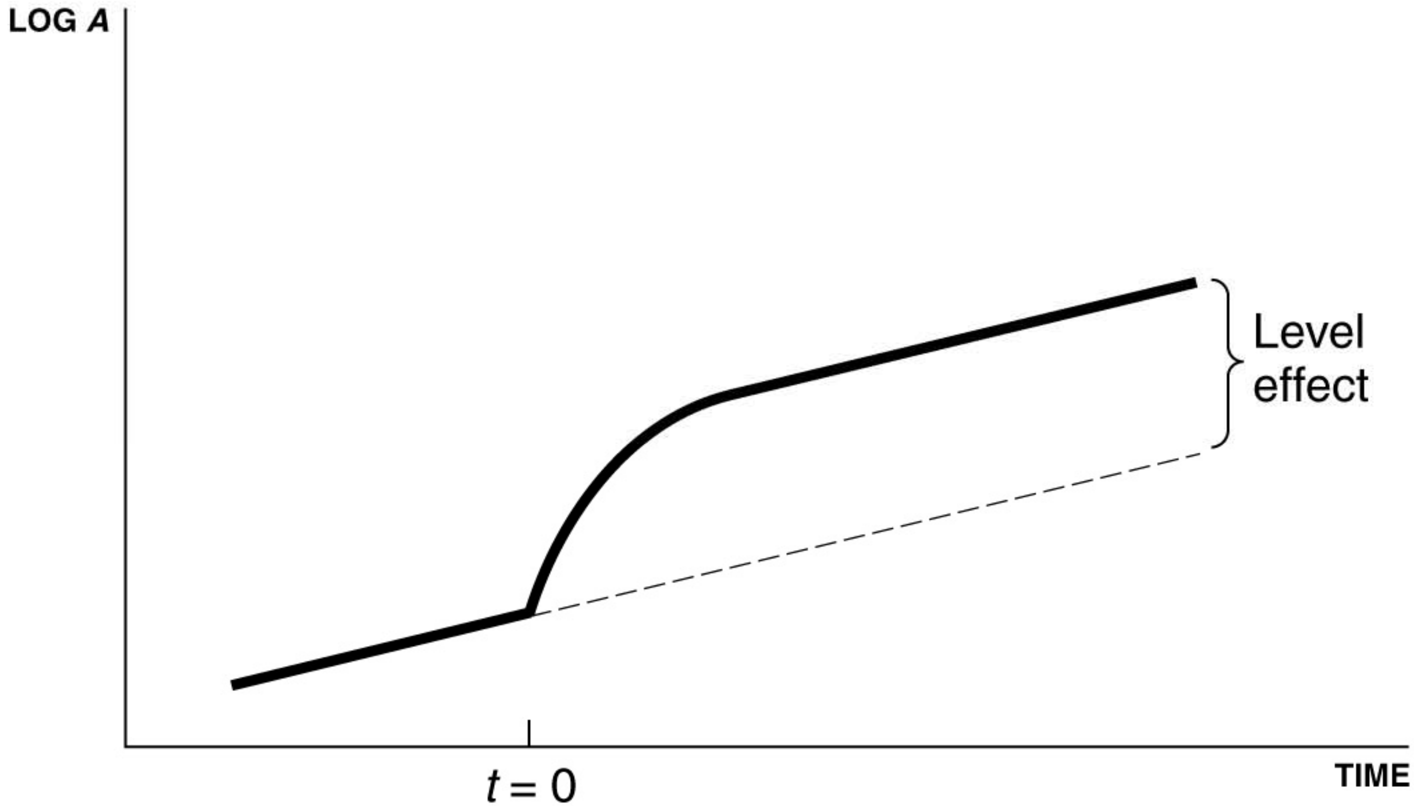

FIGURE 5.3 THE LEVEL OF TECHNOLOGY OVER TIME

Output per worker in steady state with $\lambda = 1$ and $\phi = 0$

In the steady state, $\frac{Y}{L_Y} = \left(\frac{s}{n+g_A+\delta}\right)^{\frac{1}{1-\alpha}} A(t)$. Since $L_Y = (1 - s_R)L$, $\frac{Y}{L}(t) = \left(\frac{s}{n+g_A+\delta}\right)^{\frac{1}{1-\alpha}} (1 - s_R)A(t)$. In the steady state, $\frac{\dot{A}}{A} = g_A = d\frac{s_R L}{A}$, and so $A(t) = d\frac{s_R L(t)}{g_A}$.

Thus, output per worker in the economy is

$$\frac{Y}{L}(t) = \left(\frac{s}{n+g_A+\delta}\right)^{\frac{1}{1-\alpha}} (1 - s_R)d\frac{s_R}{g_A}L(t).$$

$L(t)$ in the formula stands for the *scale effect*: a larger world economy is a richer economy (more labor, more demand for ideas (demand effect), and more ideas created (supply effect)).

s_R enters positively to reflect the idea that more researchers create more ideas and increase productivity and negatively (through the term $1 - s_R$) to reflect the idea that more researchers means less labor producing output.

Micro-foundations of the Romer model

There are 3 sectors in the economy.

1. A research sector (produces ideas).
2. An intermediate-goods sector (manufactures a capital good developed by the research sector and has an exclusive right to selling this capital good).
3. A final-goods sector (purchases the capital good and produces the final good).

The final-goods sector

The number of capital goods/ideas is A. A competitive firm producing the final good utilizes all capital goods:

$$Y = L_Y^{1-\alpha}(x_1^\alpha + x_2^\alpha + \ldots + x_A^\alpha) = L_Y^{1-\alpha}\sum_{j=1}^{A} x_j^\alpha.$$

Easier to think in terms of integration, where the number of intermediate goods is infinite and they are labeled on the real-line interval $[0, A]$.

$$Y = L_Y^{1-\alpha}\int_{j=0}^{A} x_j^\alpha dj.$$

For example, $\sum_{j=1}^{10} x_j$ and $\int_{j=0}^{10} x_j dj$ when $x_j = 100$ for all j are the same, and equal to 1,000.

The objective of the final-goods firm

$$\max_{L_Y, \{x_j\}_{j=1}^{A}} \underbrace{L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha}}_{Y} - wL_Y - \sum_{j=1}^{A} p_j x_j,$$

where p_j is the price of the j-th capital good.
At the optimum, the following $A+1$ equations should hold:

$$w = (1-\alpha) L_Y^{-\alpha} \sum_{j=1}^{A} x_j^{\alpha} = (1-\alpha)\frac{Y}{L_Y}$$

$$p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1}.$$

The second equation specifies the demand curve for capital good j.

The intermediate-goods sector

Monopolists produce the capital goods used by the final-goods sector. Patent enables each firm to produce only one capital good. The objective is:

$$\max_{x_j} \pi_j = p_j(x_j)x_j - rx_j,$$

where r is the marginal cost of producing one unit of x_j. Dropping subscript j, at the optimum, for each firm the following should be satisfied:

$$p'(x)x + p(x) - r = 0.$$

Note that $p = \frac{1}{1+\frac{p'(x)x}{p}}r$. Since $p'(x) = (\alpha - 1)\frac{p}{x}$, $p = \frac{r}{\alpha}$—a good is priced at a mark-up over marginal cost.

Production function consistent with micro-foundations

Note that $\sum_{j=1}^{A} \pi_j = \sum_{j=1}^{A} \alpha L_Y^{1-\alpha} x_j^{\alpha-1} x_j - \sum_{j=1}^{A} \alpha p_j x_j =$

$$\underbrace{\alpha\, L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha}}_{Y} - \underbrace{\alpha^2\, L_Y^{1-\alpha} \sum_{j=1}^{A} x_j^{\alpha}}_{Y} = \alpha(1-\alpha)Y.$$

If profits of each firm are equal so that $\pi_j = \pi$, then $A\pi = \alpha(1-\alpha)Y$ and $\pi = \alpha(1-\alpha)\frac{Y}{A}$. Market-cleating for the intermediate-goods sector: $K = \sum_{j=1}^{A} x_j = Ax$, or $x = \frac{K}{A}$ if $x_j = x$.

Output in the final-goods sector is:

$$Y = L_Y^{1-\alpha} \underbrace{(x^{\alpha} + x^{\alpha} + \ldots + x^{\alpha})}_{A \;\; times} = A L_Y^{1-\alpha} x^{\alpha} = A L_Y^{1-\alpha} K^{\alpha} A^{-\alpha} =$$

$K^{\alpha}(AL_Y)^{1-\alpha}$—the aggregate production function we started with in the beginning of our analysis of this chapter.

The research sector

The inventor sells his patent to the intermediate-goods firm. What is the fair price of the patent/idea, P_A? Price the patent using the *arbitrage* method.

Consider 2 options:

1. Invest the patent money at the interest rate r. One-period return is rP_A.
2. Purchase the patent. One-period return is $\pi + \dot{P}_A$.

In equilibrium, $rP_A = \pi + \dot{P}_A$, or $r = \frac{\pi}{P_A} + \frac{\dot{P}_A}{P_A}$. In steady state, r is constant and so π and P_A should grow at the same rate; since $\pi = \alpha(1-\alpha)\frac{Y}{A}$, π grows at the rate n, and so does P_A. Thus, $P_A = \frac{\pi}{r-n}$.

Wages in the model

In the final-goods sector, $w_Y = (1-\alpha)\frac{Y}{L_Y}$.

The productivity of research labor in the economy overall is defined from $\dot{A} = dL_A^{\lambda} A^{\phi}$. If a researcher ignores the economy-wide effects of his own effort, so that $\dot{A} = \bar{d}L_A$, $w_R = \bar{d}P_A$. In the equilibrium, $w_R = w_Y$, and $s_R = \frac{1}{1+\frac{r-n}{\alpha g_A}}$.

The interest rate r is obtained from
$r = \alpha p = \alpha\alpha L_Y^{1-\alpha}(\underbrace{x}_{K/A})^{\alpha} = \alpha^2 \frac{A^{1-\alpha} L_Y^{1-\alpha} K^{\alpha}}{K} = \alpha^2 \frac{Y}{K}$.

Optimal R&D

Distortions to research that cause s_R to differ from its optimal level.

1. Ideas rewarded by the stream of profits but increases in future productivity for the overall economy are not internalized into the price—too little research from a social standpoint.
2. Researchers do not "pay" for reducing productivity of others via potential duplication (if $\lambda < 1$)—too much research.
3. The "consumer-surplus effect": the gain to society is larger than the profit+consumer surplus extracted at the monopoly price—too little research.

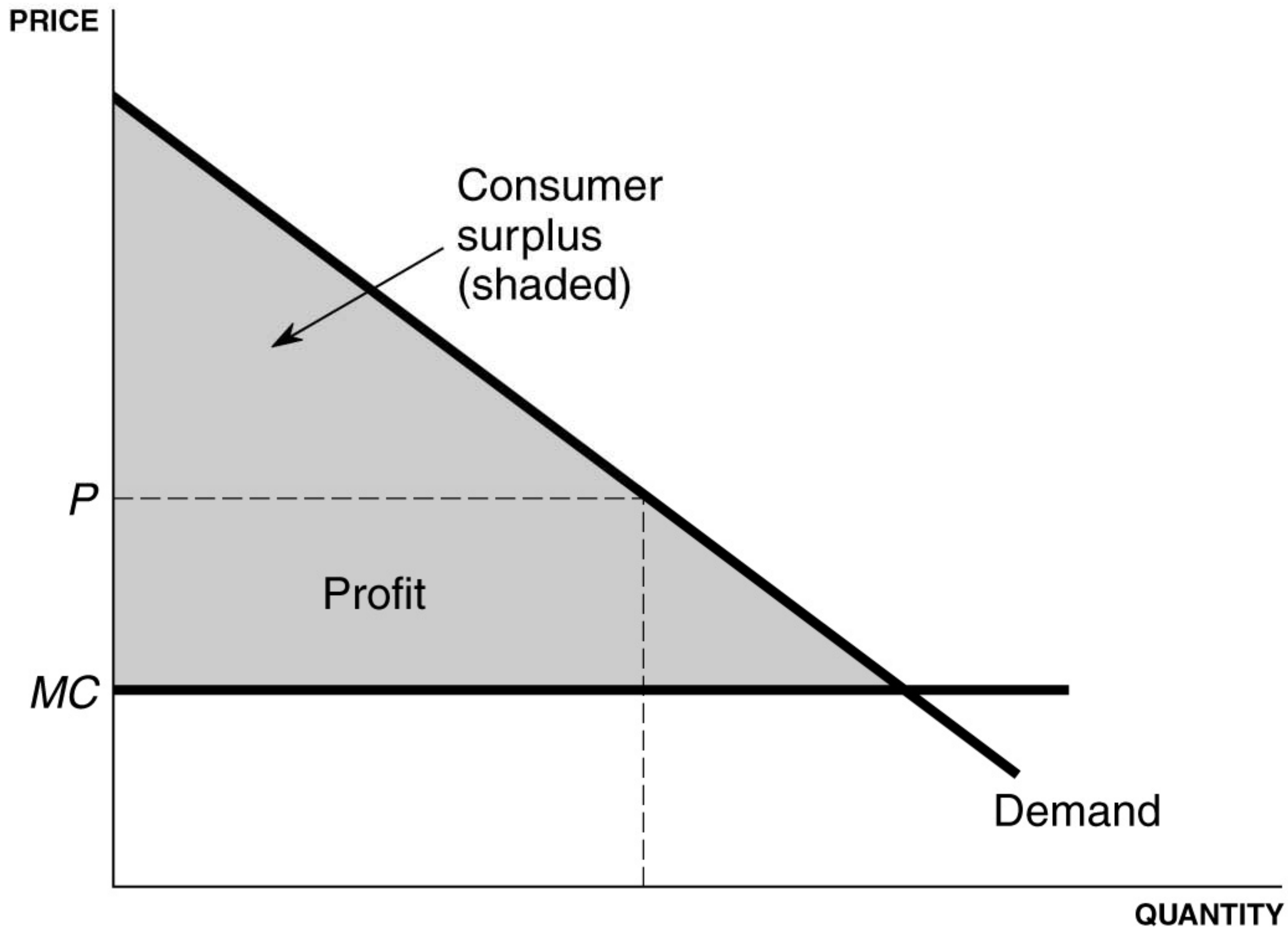

FIGURE 5.4 THE "CONSUMER-SURPLUS EFFECT"

Economic Growth, 2nd Edition