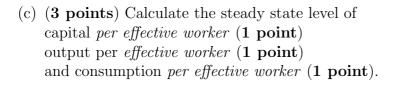
Name:		

Intermediate Macroeconomic Theory II, Winter 2011 Instructor: Dmytro Hryshko Problem set 1 (28 points). Due February 10.

- 1. (18 points) Let the economy's production function be  $Y = 4K^{1/2}(EL)^{1/2}$ . Households save 20% of their income; population growth, n, is equal to 2%; the depreciation rate,  $\delta$ , is equal to 1%; the growth rate in the efficiency of labor, g, is 1%.
  - (a) (2 **points**) Show that the aggregate production function is constant returns to scale in K and L (1 **point**), and express the production function in *per-effective-worker* terms (1 **point**).

(b) (1 point) Is production function increasing/constant/decreasing returns to scale in 3 factors of production, K, E, and L? Show how you arrived at the conclusion.



(d) (1 point) If you were a social planner who maximizes consumption per worker in the economy, what savings rate would you choose? (You need not show your calculations here if you see the answer.)

(e) (2 points) Find the golden rule level of capital per effective worker (1 point) and the corresponding output per effective worker in the golden rule steady state (1 point).

(f)	(2 points) Assume the government can tax proportionally capital and wage incomes at the rate $\tau$ and is reinvesting the tax proceedings into capital. Find the tax rate $\tau$ that will deliver the golden level of capital per effective worker you have found immediately above (continue assuming that the savings rate is 20%)
(g)	(2 points) Assume now instead that a donor outside of the economy is willing to make a gift of capital to the economy so that it reaches the golden rule steady-state. How much capital would the donor need to bring into the economy relative to what it has when the steady state is defined by the savings rate of 20%.
(h)	(1 point) What is the growth rate of total output in the steady state (on a balanced growth path, to be precise)?
(i)	(1 point) What is the growth rate of the real wage in the economy? What is the growth rate of the real interest rate?
(j)	(1 point) What is the share of capital and labor costs in total income?

(k) (2 points) Assume the economy is on a balanced growth path. Let the production function be  $Y = BK^{1/2}L^{1/2}$ , where  $B = 4E^{1/2}$ , and B is the total factor productivity. What is the contribution of the total factor productivity towards the growth in total output? That is, calculate  $\frac{\Delta B/B}{\Delta Y/Y}$ .

- 2. (10 points) Consider the Permanent Income Hypothesis we studied in class. Preferences are quadratic,  $u(c_t) = -\frac{1}{2} (\bar{c} c_t)^2$ ; planning horizon is infinite;  $\beta(1+r) = 1$ ; income stream is known as of time 0. Consider two individuals, X and Y. Individual X's income starts at 121 at time 0, drops to 110 at time 1 and stays at 110 thereafter, while individual Y's income starts at 110 at time 0, grows to 121 at time 1 and stays at 121 thereafter. The real interest rate, r, equals 10%.
  - (a) (3 points) Write down the lifetime budget constraint, and find the present discounted value of individual X's and Y's incomes.

(b) (3 points) Write down two optimality Euler conditions relating optimal consumption levels in periods 0 and 1, and optimal consumption levels in periods 1 and 2, assuming both X and Y are unconstrained in their borrowing. Find optimal consumption levels for each period  $(t=0,1,2,3,\ldots,\infty)$  for both individuals assuming that they are free to save and borrow at the interest r.

(c) (4 points) Assume that individuals are precluded from borrowing. Show how this influences the path of consumption for both individuals.