Name:______________________________________

**Intermediate Macroeconomic Theory II, Winter 2011**
**Instructor: Dmytro Hryshko**
**Problem set 1 (28 points). Due February 10.**

1. (**18 points**) Let the economy's production function be $Y = 4K^{1/2}(EL)^{1/2}$.
   Households save 20% of their income;
   population growth, $n$, is equal to 2%;
   the depreciation rate, $\delta$, is equal to 1%;
   the growth rate in the efficiency of labor, $g$, is 1%.

   (a) (**2 points**) Show that the aggregate production function is constant returns to scale in $K$ and $L$ (**1 point**), and express the production function in *per-effective-worker* terms (**1 point**).

   (b) (**1 point**) Is production function increasing/constant/decreasing returns to scale in 3 factors of production, $K$, $E$, and $L$? *Show* how you arrived at the conclusion.

(c) (**3 points**) Calculate the steady state level of
capital *per effective worker* (**1 point**)
output per *effective worker* (**1 point**)
and consumption *per effective worker* (**1 point**).

(d) (**1 point**) If you were a social planner who maximizes consumption per worker in the economy, what savings rate would you choose? (You *need not* show your calculations here if you see the answer.)

(e) (**2 points**) Find the golden rule level of capital per effective worker (**1 point**) and the corresponding output per effective worker in the golden rule steady state (**1 point**).

(f) (**2 points**) Assume the government can tax proportionally capital and wage incomes at the rate $\tau$ and is reinvesting the tax proceedings into capital. Find the tax rate $\tau$ that will deliver the golden level of capital per effective worker you have found immediately above (continue assuming that the savings rate is 20%).

(g) (**2 points**) Assume now instead that a donor outside of the economy is willing to make a gift of capital to the economy so that it reaches the golden rule steady-state. How much capital would the donor need to bring into the economy relative to what it has when the steady state is defined by the savings rate of 20%.

(h) (**1 point**) What is the growth rate of <u>total output</u> in the steady state (on a balanced growth path, to be precise)?

(i) (**1 point**) What is the growth rate of the real wage in the economy? What is the growth rate of the real interest rate?

(j) (**1 point**) What is the share of capital and labor costs in total income?

(k) (**2 points**) Assume the economy is on a balanced growth path. Let the production function be $Y = BK^{1/2}L^{1/2}$, where $B = 4E^{1/2}$, and $B$ is the total factor productivity. What is the contribution of the total factor productivity towards the growth in total output? That is, calculate $\frac{\Delta B/B}{\Delta Y/Y}$.

2. (**10 points**) Consider the Permanent Income Hypothesis we studied in class. Preferences are quadratic, $u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2$; planning horizon is infinite; $\beta(1+r) = 1$; income stream is known as of time 0. Consider two individuals, $X$ and $Y$. Individual $X$'s income starts at 121 at time 0, drops to 110 at time 1 and stays at 110 thereafter, while individual $Y$'s income starts at 110 at time 0, grows to 121 at time 1 and stays at 121 thereafter. The real interest rate, $r$, equals 10%.

   (a) (**3 points**) Write down the lifetime budget constraint, and find the present discounted value of individual $X$'s and $Y$'s incomes.

(b) (**3 points**) Write down two optimality Euler conditions relating optimal consumption levels in periods 0 and 1, and optimal consumption levels in periods 1 and 2, assuming both $X$ and $Y$ are unconstrained in their borrowing.

Find optimal consumption levels for each period ($t = 0, 1, 2, 3, \ldots, \infty$) for both individuals assuming that they are free to save and borrow at the interest $r$.

(c) (**4 points**) Assume that individuals are precluded from borrowing. Show how this influences the path of consumption for both individuals.