Chapter 5: Economic Growth II (Technological Growth)

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Further Questions

- Want to make Solow model more realistic by building in technological advancement
- What are the policies that raise the growth in output and output per worker?
- Evaluation of the Solow growth model
- Other models of economic growth

Labor-Augmenting Technological Progress

Now assume that output is a function of capital and efficient labor, $E \times L$. Production function becomes:

$$Y = F(K, EL)$$

Efficiency of labor captures society's knowledge about technology; quality of education; skills of the labor force (e.g., acquired via on-the-job training programs).

We assume that efficiency grows over time at exogenous rate g. That is, $\frac{\Delta E}{E} = g$.

Efficiency of Labor

If $g = 0.05$, then workers become 5% more efficient each year. I.e., output is increasing as if the labor force increased by additional 5%.

We usually think that technological progress is what makes labor more efficient (e.g., computerization). We say that g measures the rate of labor-augmenting technological progress.

If our production function is of the form $Y = F(E \times K, L)$, we call technological progress as capital-augmenting.

SS WITH TECHNOLOGICAL PROGRESS

Express all variables as ratios to the number of effective workers, $E \times L$: $y = Y/(EL)$, $k = K/(EL)$.

The analysis is almost the same, we make changes to our "depreciation" function and the law of motion of capital per worker (now per effective worker):

$$\Delta k = sf(k) - (\delta + n + g)k$$

Each year $(g + n) \times L_t$ new effective workers arrive. Thus, to keep capital per effective worker unchanged, we need to "hand" $(g + n) \times k$ units of capital to new and effective workers, and replace δk units of "evaporated" capital with new investment.

The law of motion of capital per effective worker

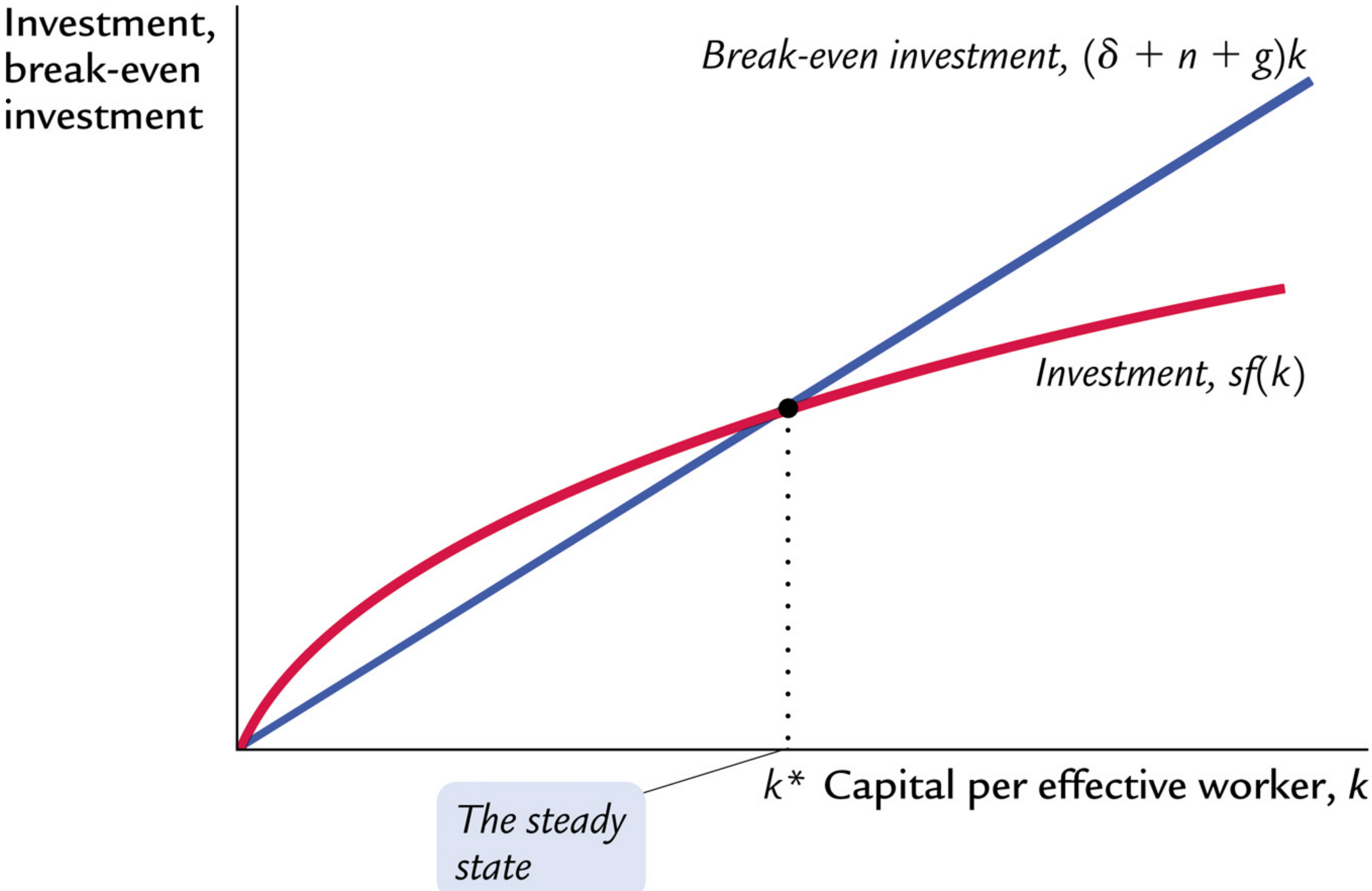
Start with $k \equiv \frac{K}{EL}$. Then,

$$\begin{aligned}
\frac{\Delta k}{k} &= \frac{\Delta K}{K} - \frac{\Delta E}{E} - \frac{\Delta L}{L} \\
&= \frac{I - \delta K}{K} - g - n \\
&= s\frac{Y}{K} - \delta - g - n \\
&= s\frac{Y/(EL)}{K/(EL)} - \delta - g - n \\
&= s\frac{y}{k} - \delta - g - n.
\end{aligned}$$

Multiplying both sides by k, we obtain

$$\Delta k = sy - (\delta + g + n)k = sf(k) - (\delta + g + n)k,$$

where $k \equiv \frac{K}{EL}$.


Investment, break-even investment
Break-even investment, (δ + n + g)k
Investment, sf(k)
k* Capital per effective worker, k
The steady state

The effects of Technological Progress

The new golden rule level of capital should now maximize $c = f(k) - (n + g + \delta)k$, and it is achieved at $MPK(k^*_{gold}) = (n + g + \delta)$.

In the SS:

- The growth of capital per worker is the growth in $k \times E$, and is equal to $0 + g = g$.
- The growth in output per worker is equal to the growth in $y \times E = g$.
- The growth in the total output is the growth in $y \times EL$ and is equal to $0 + n + g = n + g$.

Summary

What have we achieved so far?

1. Introducing a non-zero growth rate in population, our model predicts positive, sustained growth in total output at the rate n, yet does not predict the growth in output per worker.

2. Adding technological growth allows us to have positive growth both in output per worker, at the rate g, and in total output, at the rate $n + g$.

Let's now evaluate the model.

Model Performance

Does the model fit data?

- In Canadian data for the last 40 years, the growth in Y/L, and K/L is $\approx$ the same and equal to 2.1% per year.
- In the Solow model, $w = MPL = (1-\alpha)y^*_{pew}E$, where w is wage per worker, y^*_{pew} is output per effective worker in the steady state and E is the state of technology. Obviously, w worker grows at the rate g. $r = MPK - \delta = \alpha \frac{y^*_{pew}}{k^*_{pew}}$, and so the real return on physical capital should be relatively stable over the long run.
- In Canadian data, the real wage per worker has been increasing 2% per year, and real interest rate has remained constant!

To your wit: Has the Canadian economy been at its SS during the last 40 years or so?

Is Canadian Economy at the Golden Rule of Capital?

Is $MPK = n + g + \delta$?

1. Since 1950 $\frac{\Delta Y}{Y} \approx 4\%$. Thus, provided the Canadian economy is on a balanced growth path, $n + g = 0.04$.
2. $K = 3Y$ (Kaldor fact).
3. $\delta K = 0.1Y$ (reasonable assumption).
4. $MPK \times K = 0.33Y$ (fact).

Divide (3) by (2) to obtain $\delta = 0.033$. Divide (4) by (2), to obtain $MPK = 0.11$. Check it.

Now compare $MPK = 0.11$ with $n + g + \delta = 0.033 + 0.04 = 0.073$. Canadian economy under-saves (follows from the assumption of diminishing returns to capital)! Hence debates in media and politics in favor of increasing saving rate in the economy.

Growth Accounting in the Solow Model

Using Solow model, we can calculate the contribution of each factor of production towards the growth in total output.

Recall our production function is:

$$Y = F(K, L) \tag{1}$$

What is the total change in output if we change K and L by small amounts? Using a bit of calculus, it should be:

$$dY \approx \frac{\partial F(K, L)}{\partial K} dK + \frac{\partial F(K, L)}{\partial L} dL \tag{2}$$

This reads as: the total change in output is equal to the sum of the change in output due to a marginal change in capital (holding labor fixed) times the total change in capital and the change in output due to a marginal change in labor (holding capital fixed) times the total change in labor.

Growth Accounting

$\frac{\partial F(K,L)}{\partial K}$ and $\frac{\partial F(K,L)}{\partial L}$ are our familiar *MPK* and *MPL* respectively.
In a more familiar notation equation (2) can be expressed as:

$$\Delta Y = MPK \times \Delta K + MPL \times \Delta L, \tag{3}$$

which is the equation you see in the text. Now divide both sides of equation (3) by Y, to obtain:

$$\frac{\Delta Y}{Y} = \left(\frac{MPK \times K}{Y}\right)\frac{\Delta K}{K} + \left(\frac{MPL \times L}{Y}\right)\frac{\Delta L}{L}, \tag{4}$$

where we innocuously multiplied and divided two terms of summation by K and L respectively.

Growth Accounting

Now note that $\frac{MPK \times K}{Y}$ measures the share of capital income in total GDP and, for the CRS $Y = AK^{\alpha}L^{1-\alpha}$ function, it is equal to α. See my note.

Similarly, $\frac{MPL \times L}{Y}$ is the share of labor income in total income and is equal to $1 - \alpha$. Hence equation (4) becomes:

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L}. \tag{5}$$

Growth Accounting

If we take a more realistic production function $Y = AF(K, L)$, where A is the the total factor productivity (TFP) and $F(K, L)$ is the CRS, then (CHECK!)

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1 - \alpha) \frac{\Delta L}{L} + \frac{\Delta A}{A} \tag{6}$$

$\frac{\Delta A}{A}$ is also called the Solow residual since it is the contribution of TFP to output growth, not explainable by the growth in measurable factors of production. It is usually thought of as a measure of technological progress and our ignorance.

Growth Accounting for Canadian Data

In Canada, since the start of the 21st century, $\Delta Y/Y = 3\%$, $1 - \alpha = 0.67$, $\Delta L/L = 1\%$, and $\Delta K/K = 3\%$.

Hence $\Delta A/A$—the contribution of the growth in the TFP towards total output growth—is equal to 1.34%, and the TFP growth explains $1.34/3 = 45\%$ of the total output growth!

Endogenous Growth Models

- An unsatisfying feature of the Solow model is that the growth in output per worker is determined by the **exogenous** variable g.
- **Endogenous** growth models try to make g dependent on some variables observable in the economy (e.g., the share of working population engaged in research).
- Other type of endogenous models, called AK models, abandon the need for exogenous technological progress.

Endogenous Growth models—AK model

- Assume $Y = AK$ and A is some constant. Then $\Delta Y = A\Delta K$ and $\frac{\Delta Y}{Y} = \frac{A\Delta K}{Y} = \frac{\Delta K}{K} = \frac{sY - \delta K}{K} = sA - \delta$.
  Total output will grow forever if $sA > \delta$. Note that we will not even have a SS in this economy if $sA \neq \delta$. Why? Draw depreciation and investment curves to see this (assuming that $g = 0$, and thus assuming A is constant).

  Output per worker will steadily grow at the rate $\frac{\Delta Y}{Y} - \frac{\Delta L}{L} = sA - (\delta + n)$.

- Important insight of AK models is that the sustained growth in output can be generated by the economy's fundamentals—the level of technology and savings rate.

- Important feature of the production function that generates the result of sustained growth is that the returns to capital are constant, not diminishing. But...is it a reasonable assumption?