Chapter 4: Economic Growth I (No Technological Growth)

Instructor: Dmytro Hryshko

New Directions in Economic Growth

- Why are some countries rich and other poor?

Modern treatment starts with Solow (1956, 1957)—the role of accumulation of physical capital for income creation and technological growth for sustained economic growth.

Paul Romer—the economics of “ideas” and the role of human capital.

Our goal is to provide a general framework for understanding the process of growth and development.

"Summary Statistic"

- We will focus on two statistics of the average person's well-being: income and GDP per worker (*productivity measure*), and income and GDP per capita (*welfare measure*).

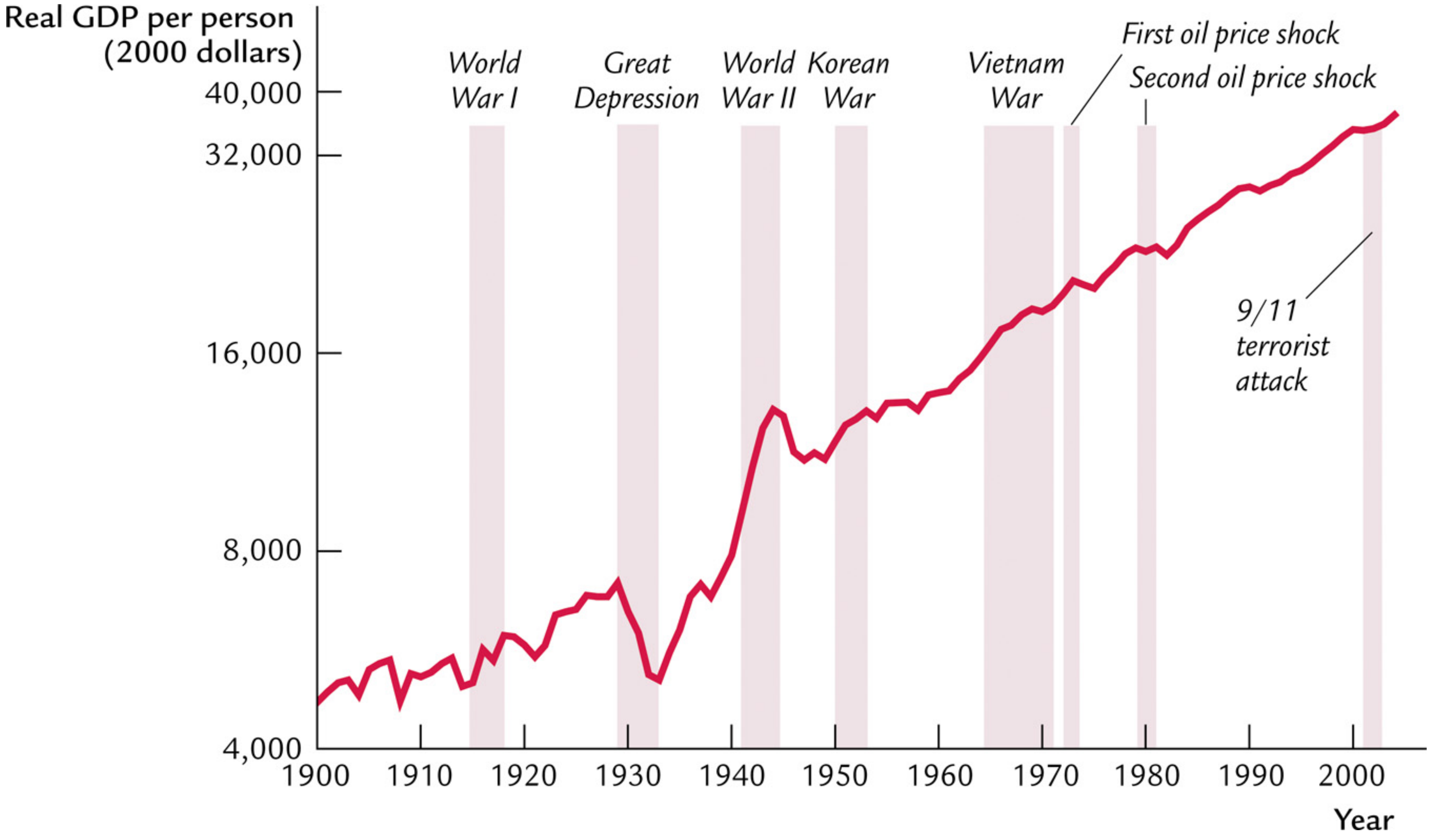
- They correlate with many other important statistics measuring well-being: infant mortality, life expectancy, consumption, etc.

Determinants of well-being and economic growth

1. We want to understand the economic dynamics, i.e., what are the factors behind the growth in real GDP.

2. What determines the economic well-being of the average person in the economy, or what are the factors behind the growth in real GDP per worker.

3. We shape our understanding using the Solow growth model.



Real GDP per person
(2000 dollars)
40,000
32,000
16,000
8,000
4,000
World War I
Great Depression
World War II
Korean War
Vietnam War
First oil price shock
Second oil price shock
9/11 terrorist attack
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
Year

Solow Growth Model

We start with a simplified variant of the Solow model.

It is a neoclassical model, where prices have no role in the determination of real quantities, like capital and output.

Assume for now there is no growth in working population and there is no technological growth. Later we'll relax these assumptions.

Facts

- **Fact 1.** There is enormous variation in incomes per capita across countries. The poorest countries have per capita incomes less than 5% of per capita incomes in the richest countries.
- **Fact 2.** Rates of economic growth vary substantially across countries.
- **Fact 3.** Growth rates are not generally constant over time. For the world as a whole, growth rates were close to zero over most of history but have increased sharply in the 20-th century. The same applies to individual countries. Countries can move from being "poor" to being "rich" (e.g., South Korea), and vice versa (e.g., Argentina).

TABLE 1.1 STATISTICS ON GROWTH AND DEVELOPMENT

	GDP per capita, 1997	GDP per worker, 1997	Labor force participation rate, 1997	Average annual growth rate, 1960–97	Years to double
"Rich" countries					
U.S.A.	$20,049	$40,834	0.49	1.4	50
Japan	16,003	25,264	0.63	4.4	16
France	14,650	31,986	0.46	2.3	30
U.K.	14,472	29,295	0.49	1.9	37
Spain	10,685	29,396	0.36	3.5	20
"Poor" countries					
China	2,387	3,946	0.60	3.5	20
India	1,624	4,156	0.39	2.3	30
Zimbabwe	1,242	2,561	0.49	0.4	192
Uganda	697	1,437	0.49	0.5	146
"Growth miracles"					
Hong Kong	18,811	28,918	0.65	5.2	13
Singapore	17,559	36,541	0.48	5.4	13
Taiwan	11,729	26,779	0.44	5.6	12
South Korea	10,131	24,325	0.42	5.9	12
"Growth disasters"					
Venezuela	6,760	19,455	0.35	−0.1	−517
Madagascar	577	1,334	0.43	−1.5	−46
Mali	535	1,115	0.48	−0.8	−85
Chad	392	1,128	0.35	−1.4	−48

SOURCE: Author's calculations using Penn World Tables Mark 5.6, an update of Summers and Heston (1991), and the World Bank's Global Development Network Growth Database, assembled by William Easterly and Hairong Yu.

Notes: The GDP data are in 1985 dollars. The growth rate is the average annual change in the log of GDP per worker. A negative number in the "Years to double" column indicates "years to halve."

The Power of Growth Rates, or Why Do We Care about Growth Rates

Time to double: Assume that y_t grows instantaneously at a constant rate g. Then $y_{t+1} = e^g y_t$. Note that this is similar to assuming that $y_{t+1} = (1+g)y_t$ for small g. Thus, $y_1 = e^g y_0$, $y_2 = e^g y_1 = e^{2g} y_0, \ldots, y_t = e^{gt} y_0$. Assume that current time is 0. What is the time needed for y_0 to double? We want to solve for t^* that satisfies $2y_0 = e^{gt^*} y_0$. Taking logs from both sides gives $\log 2 = gt^*$, or $t^* = \frac{\log 2}{g} \approx \frac{0.7}{g} = \frac{70}{100g}$, where g is expressed in percentage terms.

Thus, if $g = 0.02$ (e.g., U.S.), GDP per capita will double every $70/2 = 35$ years; if $g = 0.06$ (e.g., South Korea)—every $70/6 \approx 12$ years. If, e.g., the difference in age between the current generation of Koreans and their grandchildren is about 48 years, Korean grandchildren will be about $2^4 = 16$ times wealthier than the current generation.

Kaldor Facts

For the U.S. over the last century:

1. The real interest rate (the return on capital) shows no trend, up or down.
2. The share of labor and capital costs in income, although fluctuating, have no trend.
3. The average growth rate in output per capita has been constant and relatively constant over time, i.e., the U.S. is on a path of sustained growth of incomes per capita.

Assumptions of the Solow model

- Output is homogenous: it can be invested or consumed (think of corn: can be used as seeds for investment or consumed).
- Representative firm produces all the output in the economy, utilizing physical capital and labor.
- Representative household owns factors of production (capital and labor) and rents them to the representative firm. At time t, one unit of labor is rented at the real price $w(t)$ and one unit of capital is rented at the real price $R(t)$. Households own firms.
- Markets for labor, capital, and final good are perfectly competitive.
- Households save a constant fraction of real incomes, $s \in (0, 1)$.

Supply of Goods

- As before, supply of goods, Y is determined by the production function in the economy, $F(K, L)$.
- Assume that production function is of the CRS type. That is: $F(zK, zL) = zF(K, L) = zY$.

Example. If $Y = F(K, L) = K^{\alpha}L^{1-\alpha}$, $F(K, L)$ is constant returns to scale. Need to show that $zY = F(zK, zL)$. Note that $F(zK, zL) = (zK)^{\alpha}(zL)^{1-\alpha} = z^{\alpha}K^{\alpha}z^{1-\alpha}L^{1-\alpha} = z^{\alpha+1-\alpha}K^{\alpha}L^{1-\alpha} = zK^{\alpha}L^{1-\alpha} = zY$.

Firm's objective

At each point in time, a competitive firm maximizes profits

$$\max_{K>0, L>0} \pi = Y - wL - RK = \underbrace{F(K,L)}_{\text{revenue}} - \underbrace{(wL + RK)}_{\text{costs}}.$$

The firm's objective is to choose non-negative amounts of K and L that bring the maximum profit. Denote the profit-maximizing choice as (K^*, L^*); $F_K(K,L)$ as the marginal product of capital, and $F_L(K,L)$ as the marginal product of labor. At this choice, the following two conditions should be simultaneously satisfied:

$$F_K(K^*, L^*) = R$$
$$F_L(K^*, L^*) = w.$$

Thus, in the equilibrium,

$$\pi = F(K, L) - F_K(K, L)K - F_L(K, L)L = Y - wL - RK.$$

The assumptions of perfect competition and constant returns to scale production function imply zero profits for the representative firm. This result follows from the Euler theorem.

Thus, factor payments exhaust the total revenue, and, at each point in time:

$$Y = wL + RK.$$

Demand for Goods

- Assume that economy is closed, $NX = 0$, and $G = 0$. Then,
- $Y = C + I$. This means that output is consumed by domestic households and purchased by investors to enlarge and replace their capital stock.
- A key assumption of the Solow growth model: saving rate s—the fraction of output/income saved from additional dollar—is exogenously given (fixed).

The Law of Motion of Aggregate Capital

Aggregate capital in the economy evolves as

$$K_{t+1} = (1 - \delta)K_t + I_t,$$

where $\delta \in [0, 1]$ is the depreciation rate, and I is the aggregate investment equal to sY_t. It summarizes the process of "losing" capital in the process of production. The relationship should hold for each t. Thus, we can write

$$K_{t+1} - K_t = I_t - \delta K_t$$
$$\Delta K = I - \delta K = sY - \delta K.$$

Equilibrium

Equilibrium in the economy is defined as the path of *allocations*, C, Y, K, and *prices* w and R, given the amount of labor resources and the initial capital stock.

Per capita (worker) magnitudes

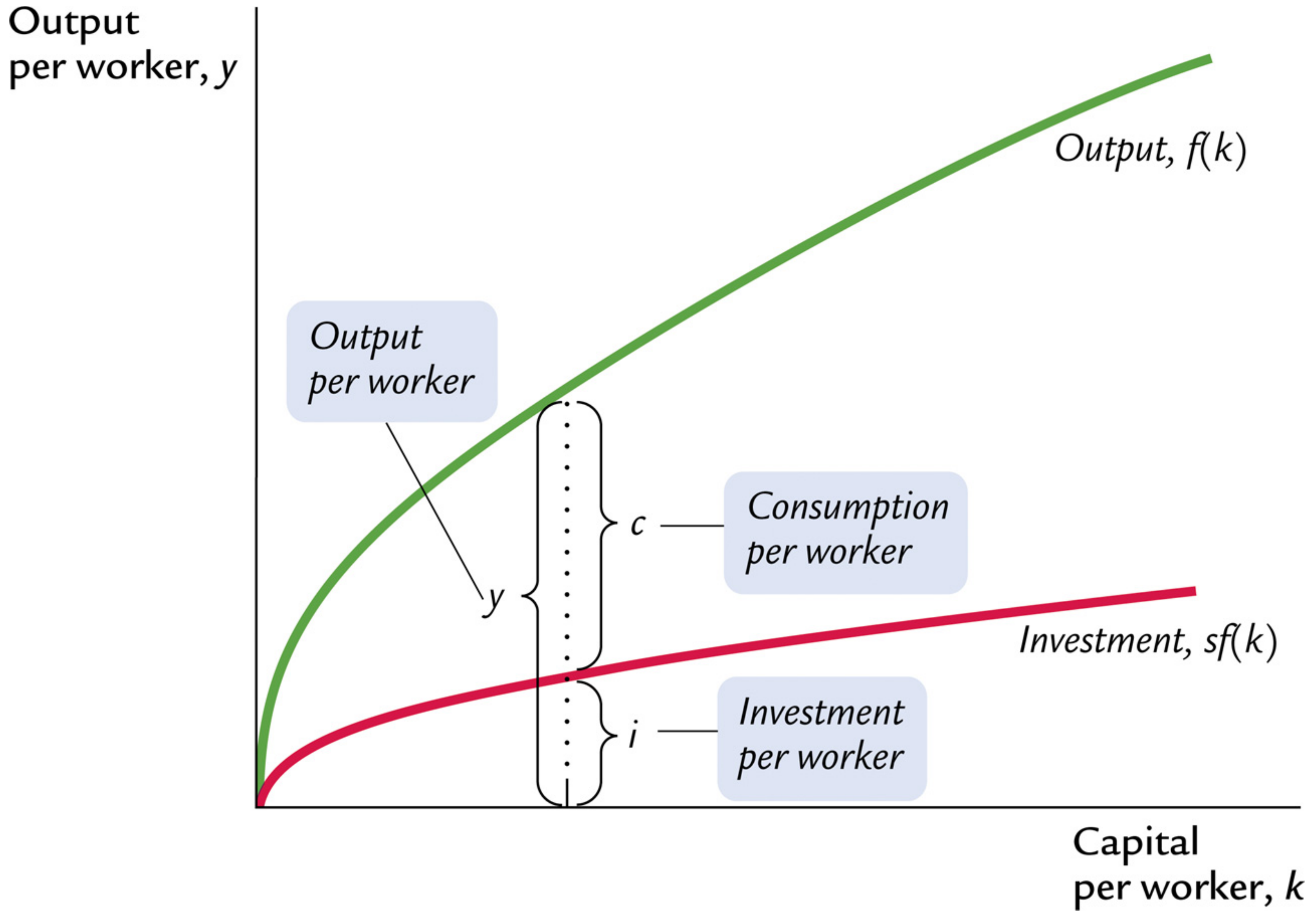
$Y = F(K, L)$ is constant returns to scale: $zY = F(zK, zL)$.

Let $z = \frac{1}{L}$. Then $F(\frac{K}{L}, \frac{L}{L}) = \frac{Y}{L}$; $F(\frac{K}{L}, 1) = \frac{Y}{L}$.

Denote $y = \frac{Y}{L}$; $k = \frac{K}{L}$. I.e., lowercase letters denote ratios of respective variables to the (working) population. Let $F(\frac{K}{L}, 1) = f(k)$.

We can rewrite production function in per capita terms as: $y = f(k)$.

Then, $c = (1 - s)y = (1 - s)f(k)$ and $y = (1 - s)y + i$, or $i = sy = sf(k)$.


Output
per worker, y
Output, f(k)
Output
per worker
y
c
Consumption
per worker
Investment, sf(k)
i
Investment
per worker
Capital
per worker, k

The Law of Motion of Capital per Worker

- Now we know that consumption per worker, output per worker, and investment per worker are functions of capital per worker only. This leads us to the question:
- How is the capital per worker determined? New capital is added each period by adding investment to the old stock of capital, and a portion of old capital wears off in the production process which leads to a lower capital stock.
- Let depreciation rate be δ. E.g., $\delta = 0.1$ means that each year 10% of capital per worker is 'lost'/wears off in production process.

Deriving the law of motion of capital per worker

Note that $k = \frac{K}{L}$. From our note,

$$\begin{aligned}
\frac{\Delta k}{k} &= \frac{\Delta K}{K} - \frac{\Delta L}{L} \\
&= \frac{I - \delta K}{K} - \frac{\Delta L}{L} \\
&= s\frac{Y}{K} - \delta - \frac{\Delta L}{L} \\
&= s\frac{Y/L}{K/L} - \delta - \frac{\Delta L}{L} \\
&= s\frac{y}{k} - \delta.
\end{aligned}$$

In the last equation, we've made use of our assumption that working population is not growing over time. Multiplying both sides of the equation by k, we obtain:

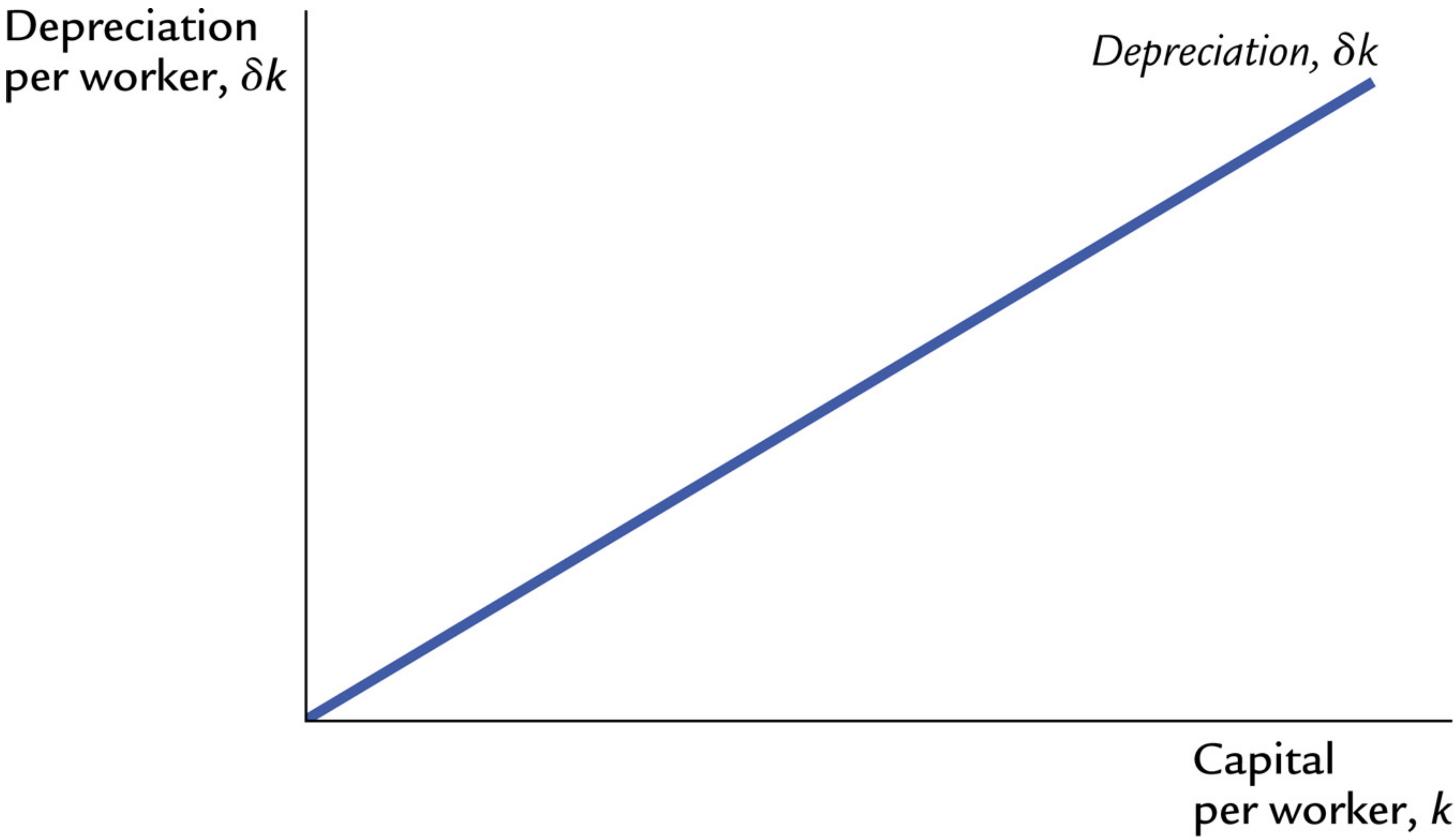
$$\Delta k = sy - \delta k = i - \delta k.$$

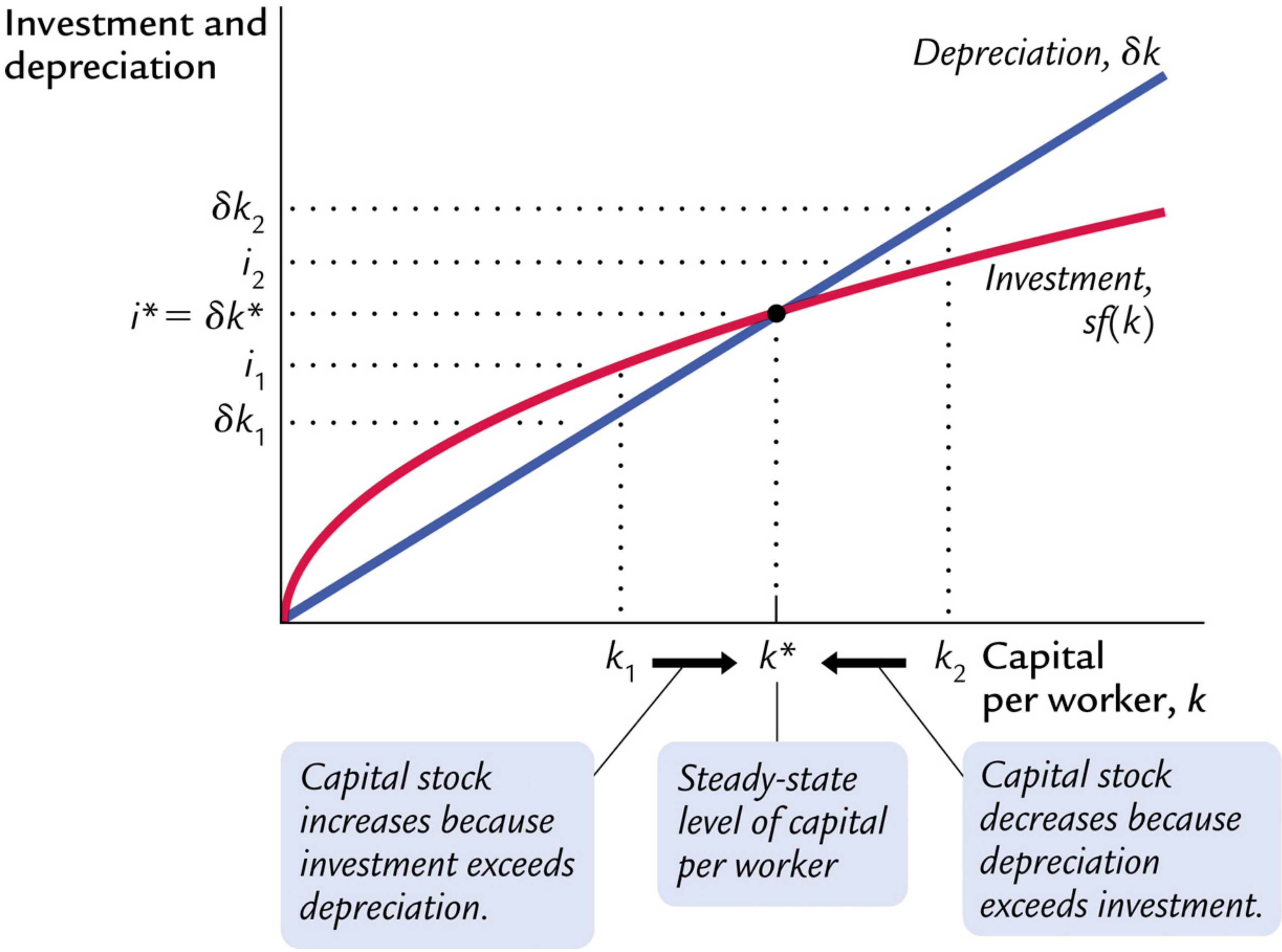
Steady-State Level of Capital

- To preserve the current capital stock unchanged from this to the next period, i.e., to have $\Delta k = 0$, we need to have investment exactly equal to depreciation. So, we need to reach such a level of capital k^* that gives us $sf(k^*) = i = \delta k^*$.

- Such a level of capital, k^* is labelled as the steady-state level of capital, and can be easily visualized at the graph.

- δk can be called the break-even investment, that is, the amount of investment needed to preserve the capital per worker at its constant level k.


Depreciation
per worker, δk
Depreciation, δk
Capital
per worker, k



Investment and depreciation
Depreciation, δk
δk2
i2
Investment, sf(k)
i* = δk*
i1
δk1
k1
k*
k2
Capital per worker, k
Capital stock increases because investment exceeds depreciation.
Steady-state level of capital per worker
Capital stock decreases because depreciation exceeds investment.

NOTES

- Steady-state (SS) level of capital per worker k^* is the one economy gravitates to in the long run regardless of its initial level of capital per worker be it above k^*, or below k^*.
- At k^*, we can determine the steady-state (long-run) value of capital per worker, the long-run value of consumption per worker, and the long run value of investment per worker.
- At steady state, output per worker and therefore standards of living stay the same over time.
- With zero population and technological growth, the growth rate of total output at the steady state is zero.

Numerical Example

Let production function be $Y = F(K, L) = K^{1/2}L^{1/2}$. Is it the CRS? Check. Let $s = 0.3$, and $\delta = 0.1$. *What does it mean?*

In per capita terms, $Y/L = (K^{1/2}L^{1/2})/L$. And so $y = K^{1/2}L^{-1/2}$, or $y = \left(\frac{K}{L}\right)^{1/2} = k^{1/2}$.

The law of motion of capital per worker is:
$\Delta k = sk^{1/2} - \delta k = 0.3k^{1/2} - 0.1k$.

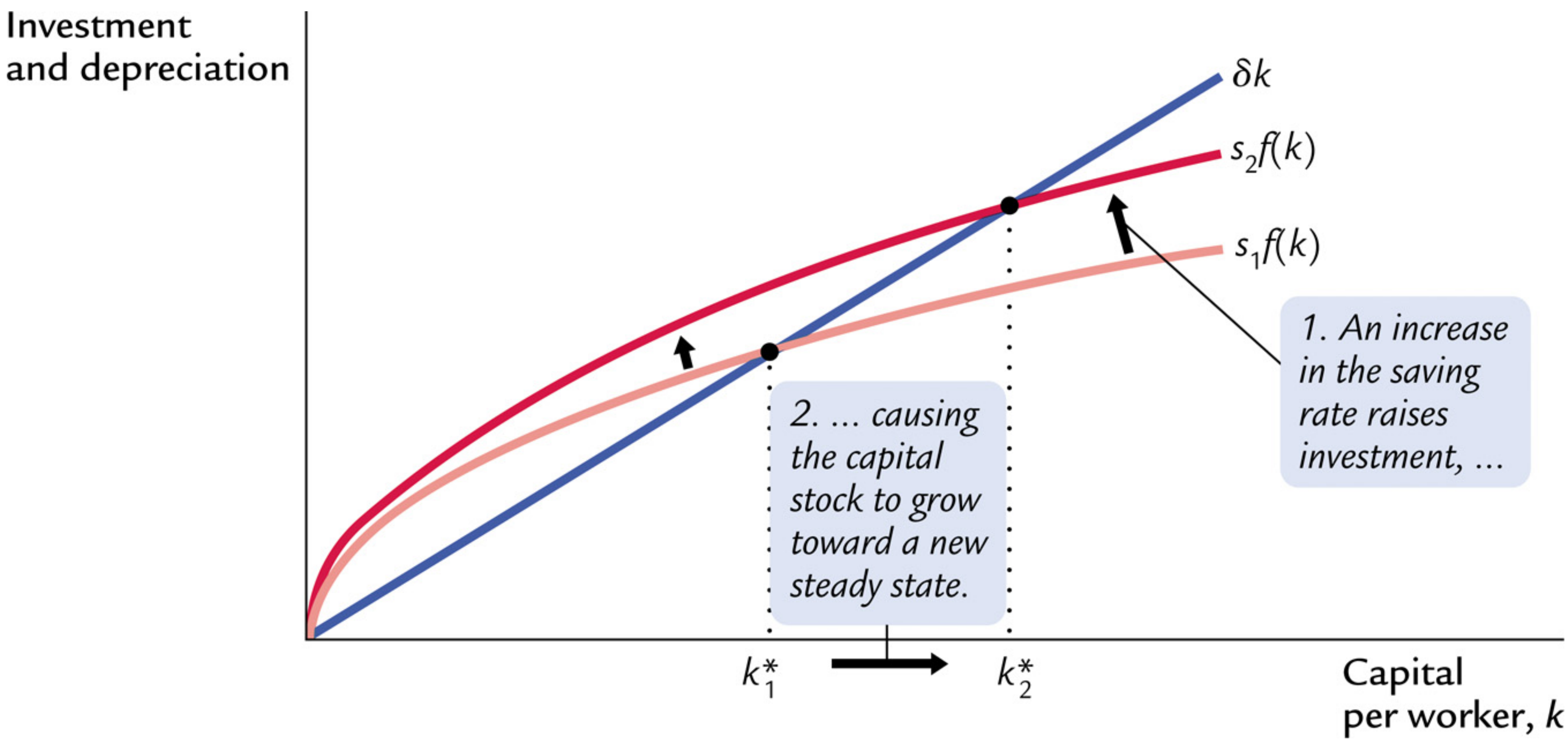
Find the SS level of capital per worker, output per worker, and consumption per worker.

At the SS, $\Delta k = 0$. Thus, the SS k^* solves:
$0.3(k^*)^{1/2} - 0.1k^* = 0$. And so $k^*/(k^*)^{1/2} = 0.3/0.1 = 3$. Thus, $k^* = 3^2 = 9$.

SS level of output per worker is just $(k^*)^{1/2} = 3$; SS level of consumption per worker is
$(1 - s)f(k^*) = 0.7(k^*)^{1/2} = 0.7 \times 3 = 2.1$

The Effect of Savings on Growth

- If savings rate increases from s_1 to s_2, we know that investment will be higher for any level of capital per worker, and this holds for the old SS k^* as well.
- At the old SS k^*, investment will be higher than depreciation and so capital per worker will grow until it reaches the new SS.
- The path of the economy from the old SS to the new SS is called a transitional path of the economy.



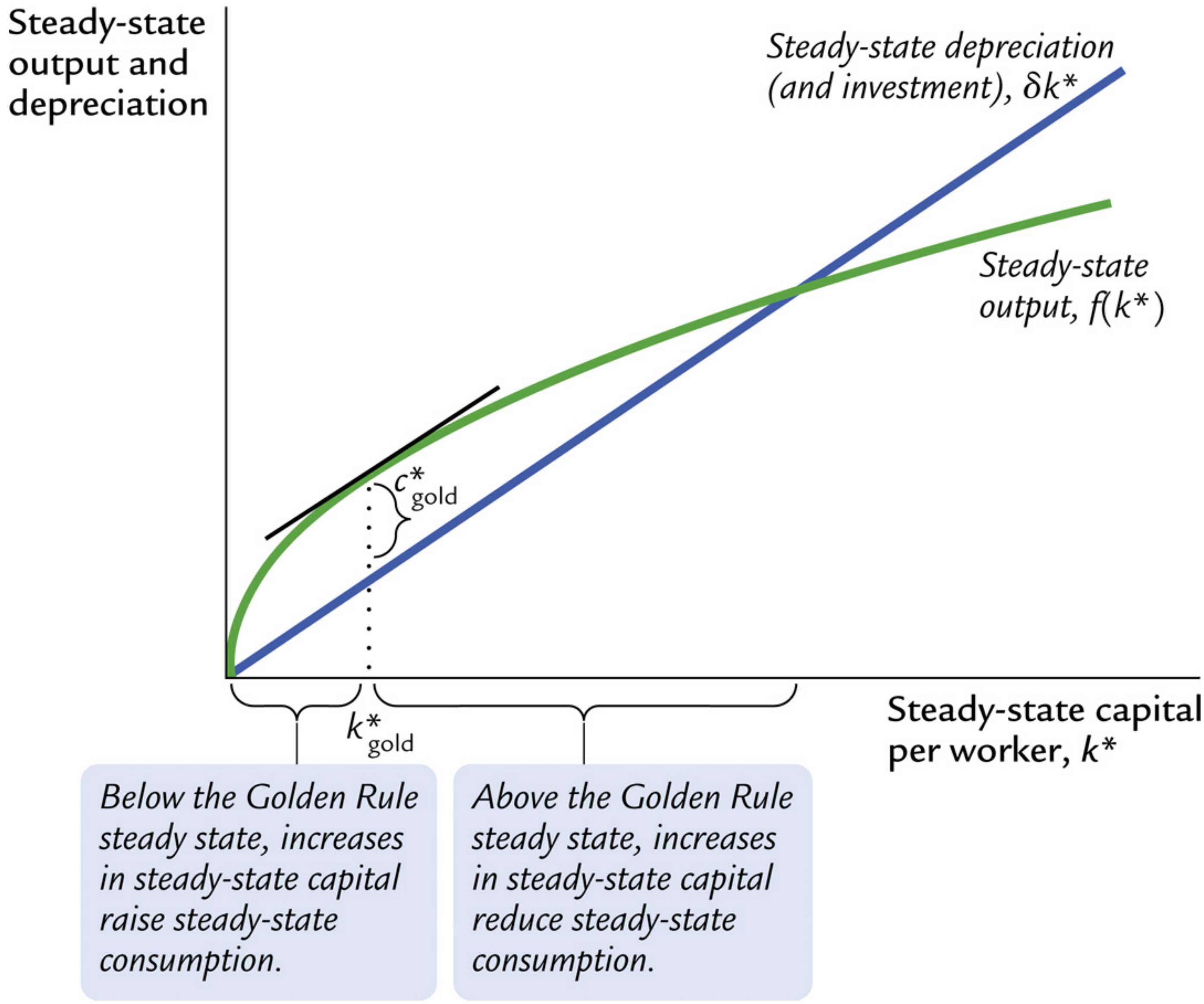
Investment and depreciation
δk
$s_2f(k)$
$s_1f(k)$
1. An increase in the saving rate raises investment, ...
2. ... causing the capital stock to grow toward a new steady state.
k_1^*
k_2^*
Capital per worker, k

The Effect of Savings on Growth

- Note that at the new SS, i is higher and y is higher. Yet the growth of Y and y are zero, as they are at the old SS. The growth rates are greater than zero during the transition from the old to the new SS!
- It is one of many rationales to encourage savings in the economy—to promote higher levels of output and standards of living.

Golden Rule of Capital

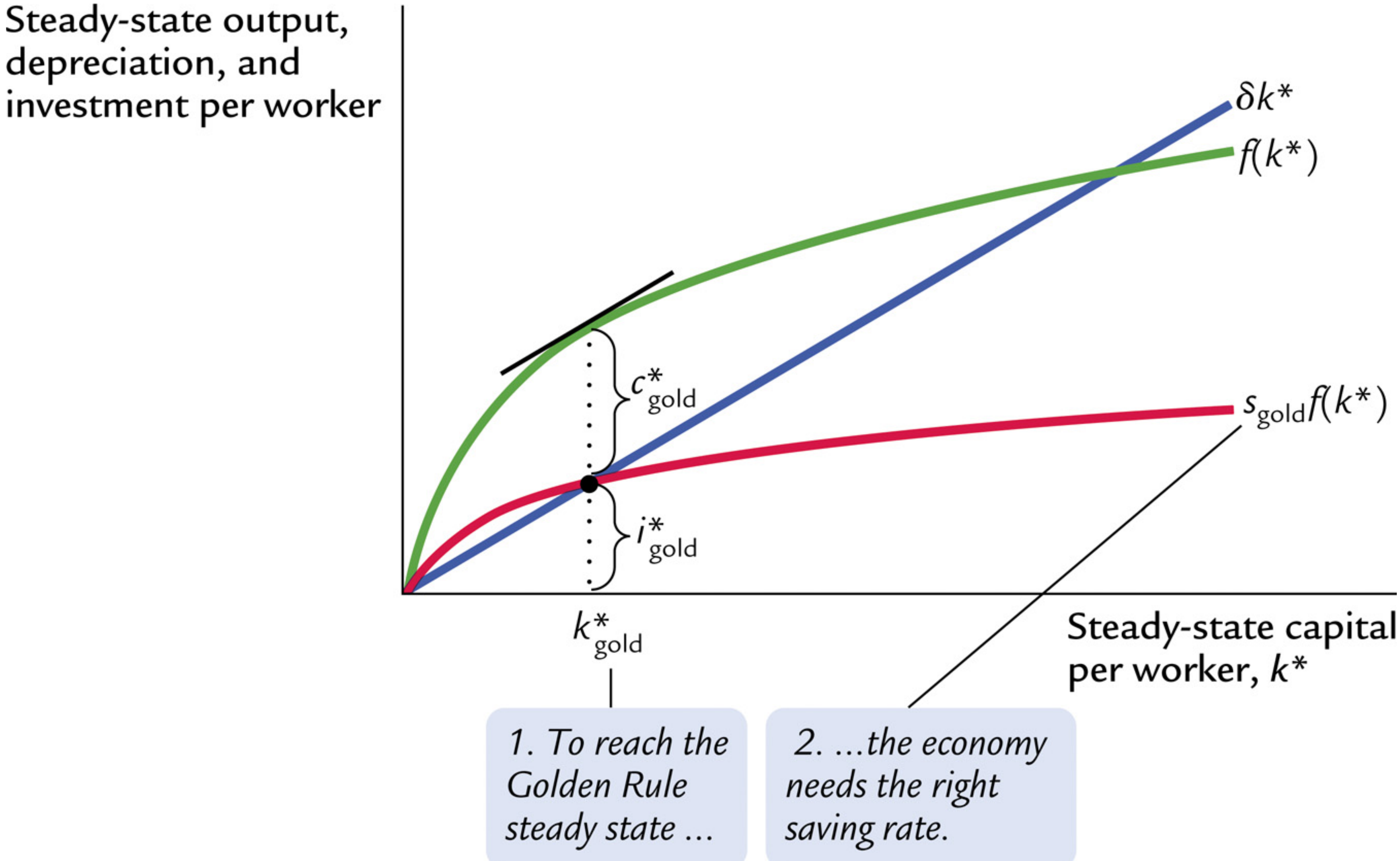
- We usually care about consumption which is a better measure of welfare than output.
- Imagine a social planner who wants to maximize consumption of the average worker, and who can set s and k. What would he choose?
- Planner wants to maximize $c = y - i = f(k) - sf(k)$. At the SS, $c^* = f(k^*) - \delta k^*$ since $sf(k^*) = \delta k^*$ at the SS.
- The maximum of c occurs at the level of k^*_{gold} where $MPK(k^*_{gold}){=}\delta$.
- More formally, k^*_{gold} solves $\frac{\partial}{\partial k}[c] = 0$, which happens at $\frac{\partial}{\partial k}[f(k^*_{gold})] - \frac{\partial}{\partial k}[\delta k^*_{gold}] = 0$, or at $f'(k^*_{gold}) = \delta$


Steady-state output and depreciation
Steady-state depreciation (and investment), δk*
Steady-state output, f(k*)
c*gold
k*gold
Steady-state capital per worker, k*
Below the Golden Rule steady state, increases in steady-state capital raise steady-state consumption.
Above the Golden Rule steady state, increases in steady-state capital reduce steady-state consumption.

Numerical Example

To reach the golden rule level of capital per worker, the planner needs to induce the saving rate s that will support this SS level of capital.

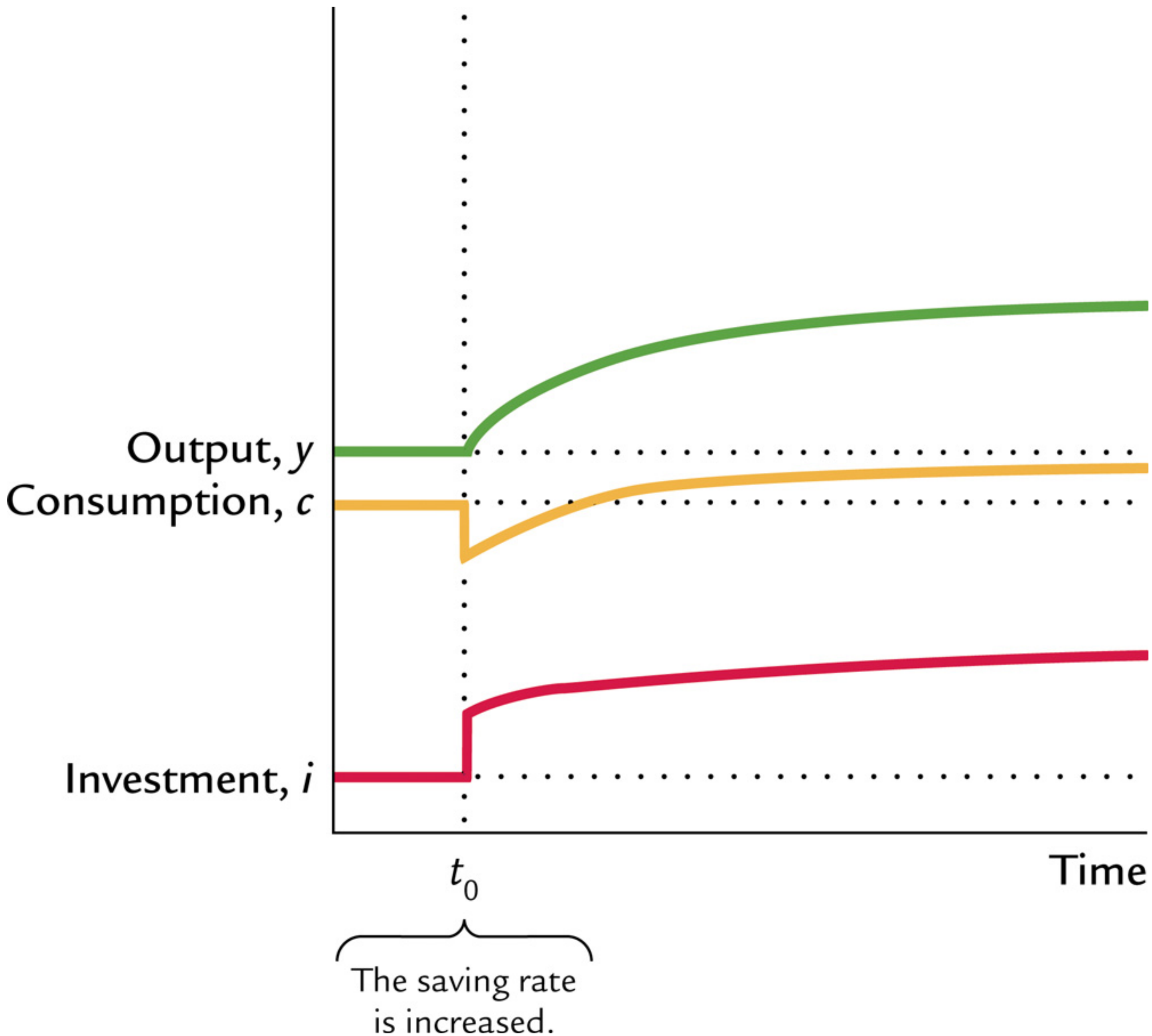
- Find k^*_{gold} and s_{gold} for the example above.
- At the SS: $s(k^*_{gold})^{1/2} = 0.1k^*_{gold}$. Thus, $s = 0.1 \times (k^*_{gold})^{1/2}$ (1).
- We also know that $MPK(k^*_{gold}) = \delta$.
- $MPK = f'(k)$. How to find $f'(k)$? For a power function, $f(x) = x^\alpha$, $f'(x) = \alpha x^{\alpha-1}$.
- Thus for our example $MPK = 1/2(k^*_{gold})^{-1/2}$, or $1/2 \times 1/(\sqrt{k^*_{gold}})$. And so... $\sqrt{k^*_{gold}} = 5$, and $k^*_{gold} = 25$. From (1), $s_{gold} = 0.1 * 5 = 0.5$.



Steady-state output, depreciation, and investment per worker
δk^*
$f(k^*)$
c^*_{gold}
$s_{gold}f(k^*)$
i^*_{gold}
k^*_{gold}
Steady-state capital per worker, k^*
1. To reach the Golden Rule steady state ...
2. ...the economy needs the right saving rate.

Transition to the Golden rule steady state

What if the economy has higher or lower savings rate than the one generating the golden rule level of capital and maximum consumption per worker?


Output, y
Consumption, c
Investment, i
t_0
Time
The saving rate is increased.

Relaxing the assumption of no population growth

Let's say that population in the economy grows n per cent per year, that is, $\frac{\Delta L}{L} = n$.

- We know that production and investment functions do not change, yet the "depreciation" function changes.
- The capital per worker falls not only because the stock of capital wears off but also because, with positive population growth, there are more workers using the stock of capital.
- We can write the law of motion of capital as $\Delta k = i - (n + \delta)k$, or $\Delta k = sy - (n + \delta)k = sf(k) - (n + \delta)k$.

The law of motion of capital per worker with positive population growth

Recall: $k = \frac{K}{L}$. From our note,

$$\begin{aligned}
\frac{\Delta k}{k} &= \frac{\Delta K}{K} - \frac{\Delta L}{L} \\
&= \frac{I - \delta K}{K} - \frac{\Delta L}{L} \\
&= s\frac{Y}{K} - \delta - \frac{\Delta L}{L} \\
&= s\frac{Y/L}{K/L} - \delta - \frac{\Delta L}{L} \\
&= s\frac{y}{k} - (n + \delta).
\end{aligned}$$

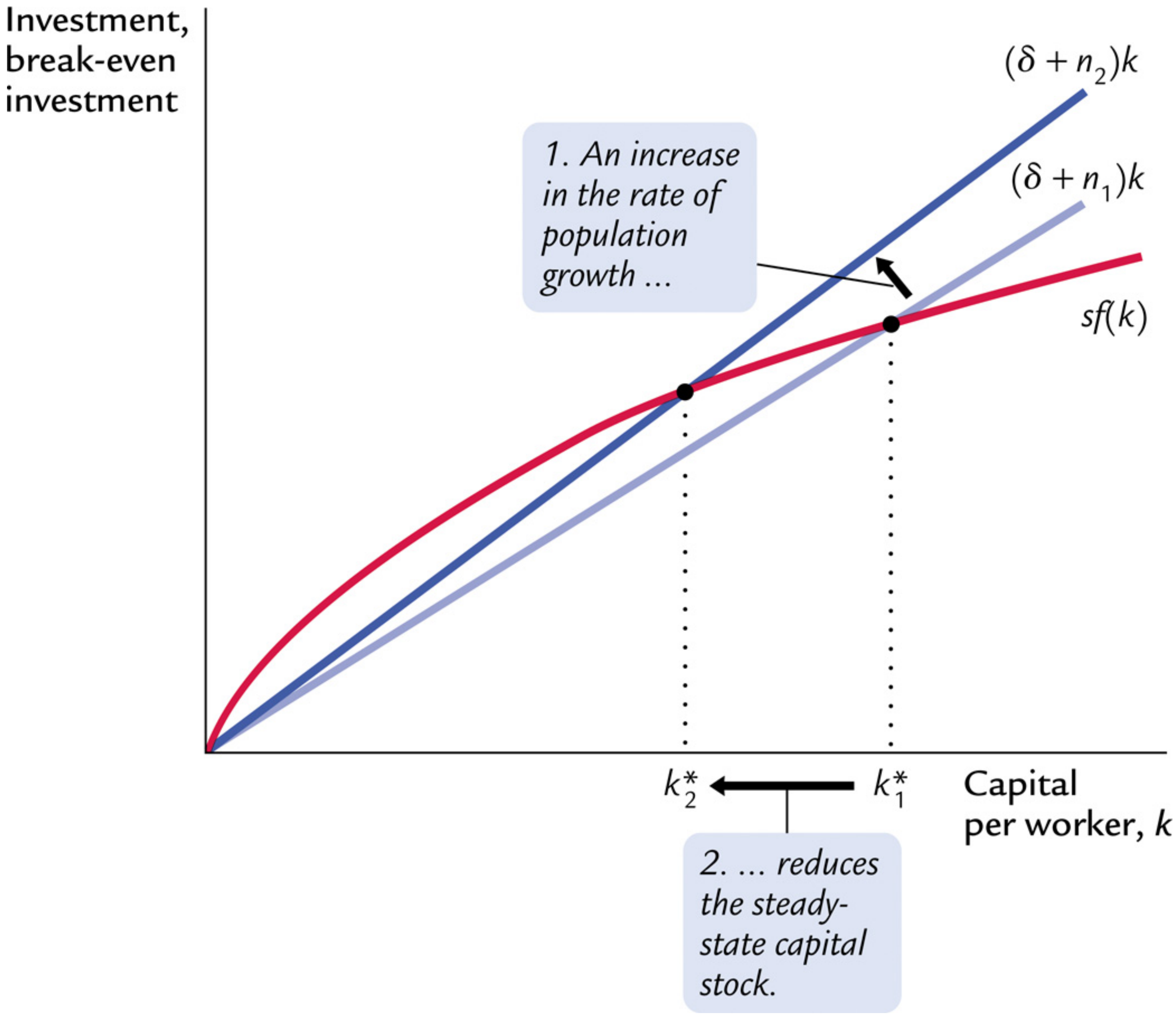
Multiplying both sides of the equation by k, we obtain:

$$\Delta k = sy - (n + \delta)k = i - (n + \delta)k = sf(k) - (n + \delta)k.$$

Steady state with positive population growth

Economy will settle down at k^* such that

$$\Delta k = \underbrace{sf(k^*)}_{\text{steady state inv.}} - \underbrace{(n+\delta)k^*}_{\text{``effective depreciation''}} = 0.$$



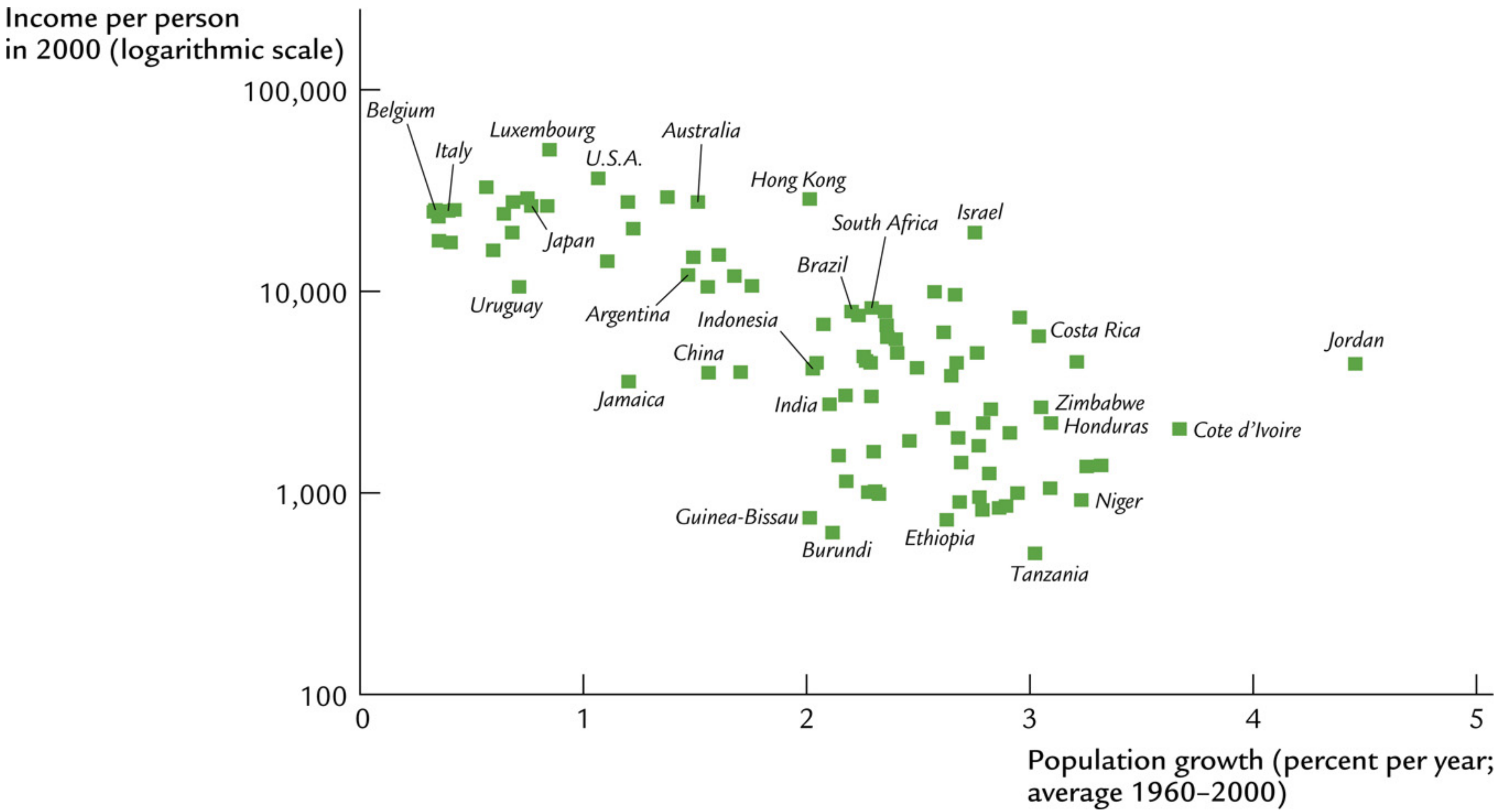
Investment, break-even investment
$(\delta + n_2)k$
1. An increase in the rate of population growth ...
$(\delta + n_1)k$
$sf(k)$
k_2^*
k_1^*
Capital per worker, k
2. ... reduces the steady-state capital stock.

Effects of Population Growth

If population growth changes from zero to some positive n:

- New steady-state k^*, and y^* are lower.
- Golden rule level of k is determined by maximizing $c = f(k^*_{gold}) - (n+\delta)k^*_{gold}$. WHY?

  It is determined by solving $MPK(k^*_{gold}) = n + \delta$.
- E.g., if n=0.05 in our example, new k^*_{gold} is equal to 11.11. Check! Determine the golden rule savings rate.



Income per person
in 2000 (logarithmic scale)
100,000
10,000
1,000
100
Belgium
Italy
Luxembourg
U.S.A.
Australia
Hong Kong
South Africa
Israel
Japan
Brazil
Uruguay
Argentina
Indonesia
Costa Rica
Jordan
China
Jamaica
India
Zimbabwe
Honduras
Cote d'Ivoire
Guinea-Bissau
Niger
Burundi
Ethiopia
Tanzania
0
1
2
3
4
5
Population growth (percent per year;
average 1960–2000)

Add-on

Note that in the steady state of an economy with positive population growth:

1. y does not grow. At odds with data since standards of living are changing over time.

2. Y grows at the rate n. WHY?

Next, we will try to fix 1.