

# CHAPTER 3: NATIONAL INCOME (CREATION AND DISTRIBUTION)

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# PLAN

## Plan:

- 1 Show what determines supply of goods and services in the economy
- 2 Show how the income from the goods sold is distributed in the economy
- 3 Show how demand for goods and services is determined. Determinants of aggregate consumption, investment, and government spending
- 4 Show how supply of and demand for goods in the economy are brought into the equilibrium

# DETERMINANTS OF AGGREGATE SUPPLY

To produce output we need capital ( $K$ ) and labor ( $L$ ). They are called factors of production. (Other examples of factors of production: land, natural resources, health.)

We utilize factors of production and some specific technology to produce final goods.

The way of transforming  $K$  and  $L$  into the final output  $Y$  is called a production function.

Production function can be expressed as:

$$Y = F(K, L)$$

Specific examples of production functions:  $Y = K^{1/3}L^{2/3}$ ;  
 $Y = KL$ ;  $Y = \min\{K, L\}$ .

# AGGREGATE SUPPLY IN THE LONG RUN (LR)

Let  $\bar{X}$  mean that variable  $X$  is fixed (pre-determined, exogenous).

Assume that we are in the LR and that available factors of production are fixed and fully utilized, i.e.  $K_{LR} = \bar{K}$ , and  $L_{LR} = \bar{L}$ .

By definition,

$$Y_{LR} = \bar{Y} = F(K_{LR}, L_{LR}) = F(\bar{K}, \bar{L})$$

# DISTRIBUTION OF AGGREGATE INCOME

Revenue from the output sold is distributed by paying:

- The wage bill,  $W \times L$ ;
- The rental cost of capital,  $R \times K$ ;
- Reward for the entrepreneurs' abilities—profit.

$W$ —(nominal) wage rate (wage per hour);  $R$ —the rental price of one unit of capital.

Firm's demand for factors of production and its profit,  $\pi$ , are determined by solving the maximization problem:

$$\max \pi = P \times Y - W \times L - R \times K$$

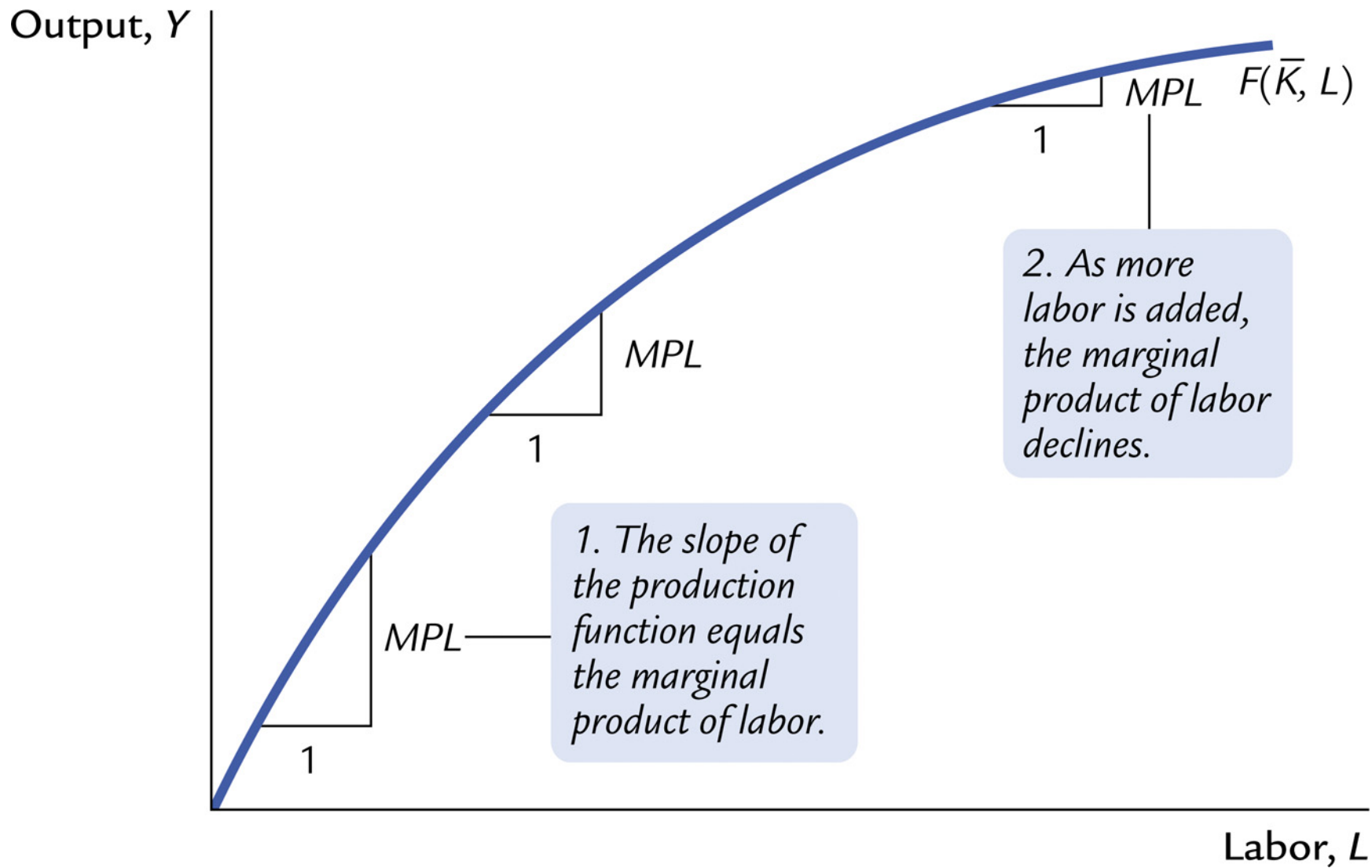
# DEMAND FOR LABOR AND CAPITAL

Assume that markets for labor and capital, and for final goods are perfectly competitive.

It means that firms are price takers in the goods and factor markets: their decisions do not affect the market prices since they are too small relative to the market.

Define the marginal product of labor, MPL, as the additional product produced by one more worker. Similarly define the marginal product of capital, MPK.

We usually assume, holding constant the amount of capital utilized, that MPL is decreasing in the amount of labor employed. Similarly, MPK is decreasing in the level of capital utilized.



## DEMAND FOR LABOR AND CAPITAL—CONTD.

Firm's choice of labor: Increase  $L$  until the value of extra output produced by an additional worker is equal to the extra costs of hiring this worker. That is firm chooses  $L^*$  such that:

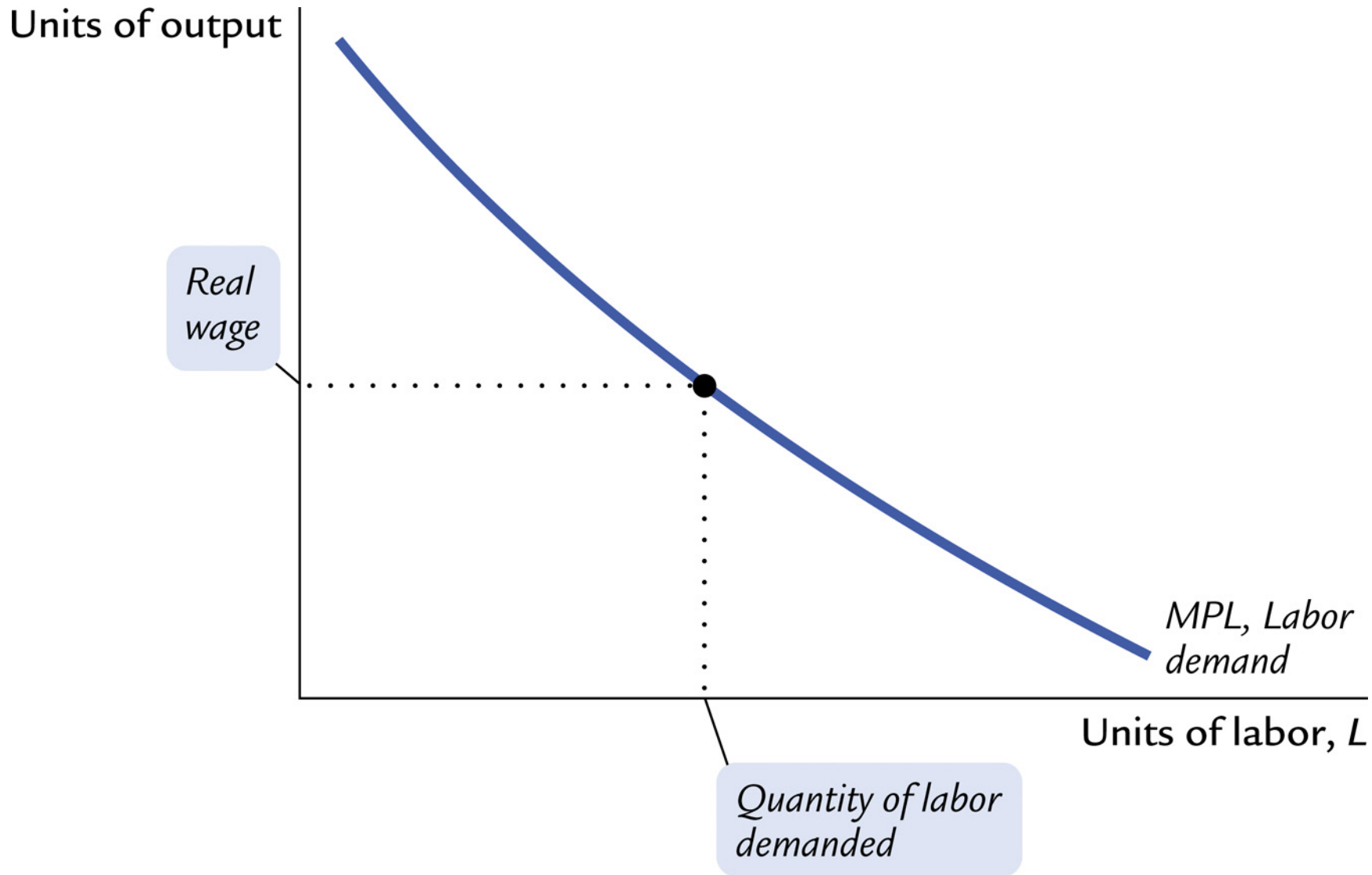
$$P \times MPL(L^*) - W = 0$$

In other words, choose  $L^*$  such that  $MPL(L^*) = \frac{W}{P}$ .

Note that both  $W$  and  $P$  are exogenous, and so  $MPL$  is the firm's labor demand.  $\frac{W}{P}$  is called real wages. E.g., if  $W=10$  doll./hour and  $P=5$  doll./book, then the real wage is 2 books/hour.

Similarly, choose capital  $K^*$  such that  $P \times MPK(K^*) = R$ .





# FOCs OF PROFIT MAXIMIZATION PROBLEM

More rigorously, firm maximizes profit at  $K^*$  and  $L^*$  that solve 2 first order conditions (FOCs):

$$\frac{\Delta\pi}{\Delta L} = P \times \frac{\Delta F(K^*, L^*)}{\Delta L} - W = 0$$

$$\frac{\Delta\pi}{\Delta K} = P \times \frac{\Delta F(K^*, L^*)}{\Delta K} - R = 0$$

These are the same the same conditions as above, with  $MPL(L^*) = \frac{\Delta F(K^*, L^*)}{\Delta L}$ , and  $MPL(K^*) = \frac{\Delta F(K^*, L^*)}{\Delta K}$ .

# FACTOR PAYMENTS AND ECONOMIC PROFIT

Real profit (relative to price) is:

$$Y - \frac{W}{P}L - \frac{R}{P}K = Y - MPL \times L - MPK \times K$$

where  $L = L^*$ ,  $K = K^*$ ,  $MPL = MPL(L^*, K^*)$ ,  
 $MPK = MPK(L^*, K^*)$ , and the second equality comes from the  
profit maximization.

Thus, real income  $Y$  is distributed as:

$$F(K, L) = Y = MPL \times L + MPK \times K + Profit$$

## CRS Technology

Under constant returns to scale (CRS) technology, profit is ZERO!  
And so...

$$Y = MPL \times L + MPK \times K \quad (\text{Euler Theorem})$$

What is the CRS production function?

CRS means that magnifying factors of production by factor  $z$  leads to the magnification of output by the same factor ( $z$ ).

Formally:

$$F(zK, zL) = zF(K, L) = zY$$

Example: Is  $Y = F(K, L) = K + L$  a CRS? Check if

$$F(zL, zK) = zY.$$

$F(zL, zK) = zK + zL = z(K + L) = zF(K, L) = zY$ . YES, it is CRS.

Check if  $Y = K^{1/3}L^{2/3}$ ;  $Y = KL$ ;  $Y = \min\{K, L\}$  are CRS functions.

## PROOF OF THE EULER THEOREM—OPTIONAL

Assuming constant returns to scale,

$$zF(K, L) = F(zK, zL) \quad (1)$$

Define  $F_K$ , and  $F_L$  as the partial derivatives of the production function with respect to capital and labor (i.e.,  $MPK$  and  $MPL$ );  $zK$  as  $K'$ , and  $zL$  as  $L'$ . Differentiate the above identity with respect to  $z$  to obtain:

$$\begin{aligned} \frac{\partial}{\partial z}[zF(K, L)] &= F(K, L) = \frac{\partial}{\partial z}[F(K', L')] = \\ & \frac{\partial K'}{\partial z} \times F_{K'}(K', L') + \frac{\partial L'}{\partial z} \times F_{L'}(K', L') = \\ & K \times F_{K'}(K', L') + L \times F_{L'}(K', L') \end{aligned} \quad (2)$$

Where the third equality is an application of the chain rule of differentiation. The equalities 1 and 2 hold for any  $z$ , and so for  $z=1$ . Thus, the Euler theorem reads:

$$F(K, L) = Y = K \times F_K(K, L) + L \times F_L(K, L) = K \times MPK + L \times MPL$$

# AGGREGATE DEMAND

We have just established that:

- 1 Aggregate supply is determined by the availability of the factors of production and technology that transforms them into final goods.
- 2 Real profits are exhausted by the payments to labor and capital, used in production.
- 3 We are done with the supply side of the economy. Now we switch to the demand side.

Further questions:

- What determines the components of aggregate demand?
- What ensures equilibrium in the goods markets? I.e., what ensures that aggregate demand equals to aggregate supply?

# AGGREGATE DEMAND

Recall the national accounts identity:

$$Y = C + G + I + NX$$

- Assume economy is closed, i.e. no imports and exports. Then,  $Y = C + I + G$ .
- Assume government purchases are fixed, i.e.  $G = \bar{G}$ .
- Need to identify the determinants of  $C$  and  $I$ .

## CONSUMPTION EXPENDITURES $\approx$ 59% OF GDP

Assume that aggregate consumption is the function of disposable income, i.e. the after-tax income.

$$C = C(Y - T) = C(Y^d)$$

Where  $Y$  is the income of households (equal to GDP, WHY?),  $T$  are aggregate taxes minus aggregate transfers ( $Tr$ ), and  $Y^d$  is the aggregate disposable income.

This consumption function is usually labelled the Keynesian consumption function.

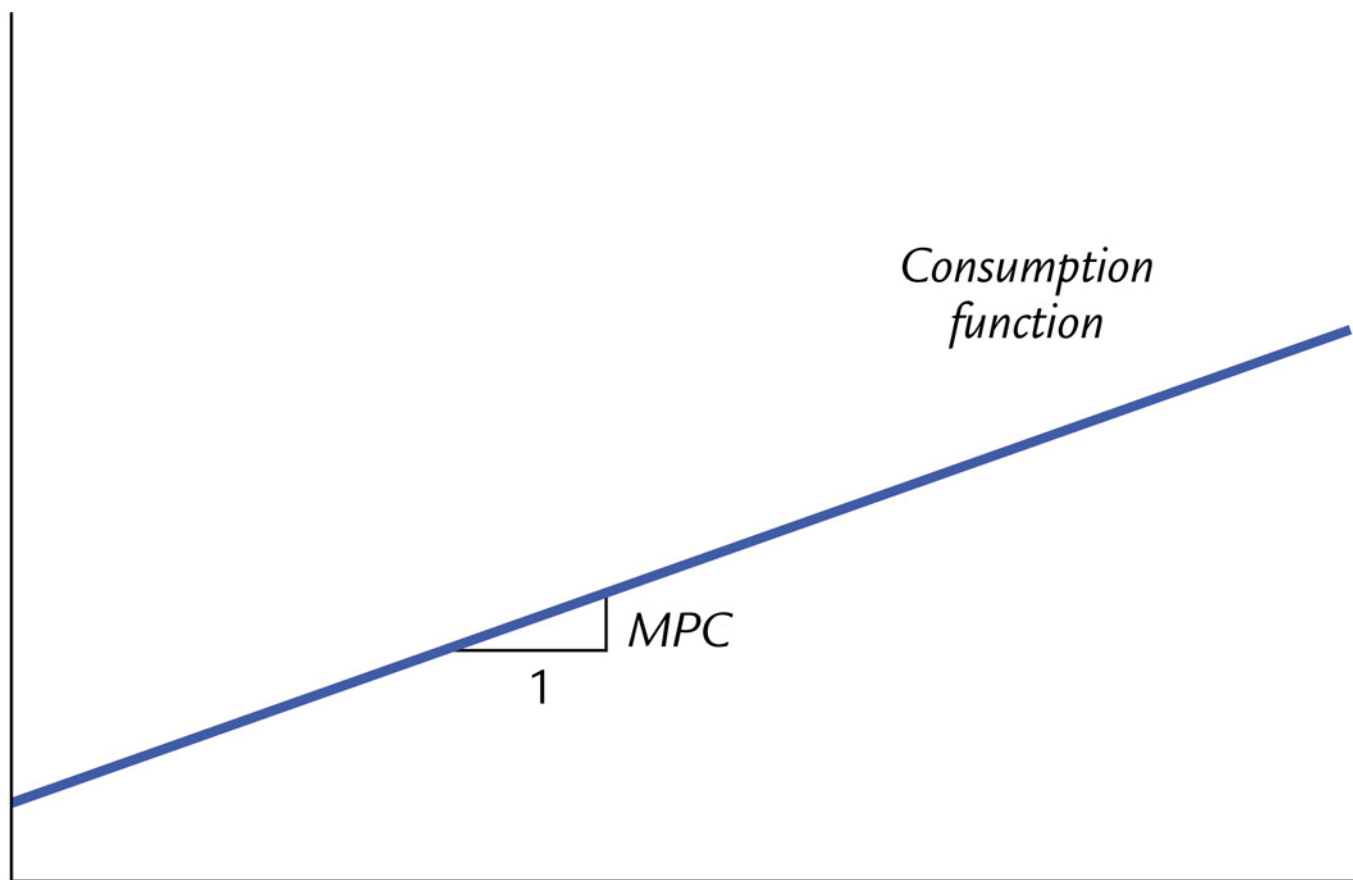
A key parameter in macro is the marginal propensity to consume ( $MPC$ ): it measures extra consumption done from one additional dollar of income.

$MPC = \frac{\Delta C}{\Delta Y^d}$ .  $MPC$  is bounded between zero and one.

Consumption function curve is upward sloping!



Consumption,  $C$



*Consumption  
function*

$MPC$

1

Disposable income,  $Y - T$

## INVESTMENT DEMAND $\approx$ 17% OF GDP

Demand for investment is a function of the interest rate which measures the price of capital.

Investor decides whether to invest on the basis of the real interest rate, which is the nominal interest rate (we are used to) adjusted for inflation.

$$r = i - \pi,$$

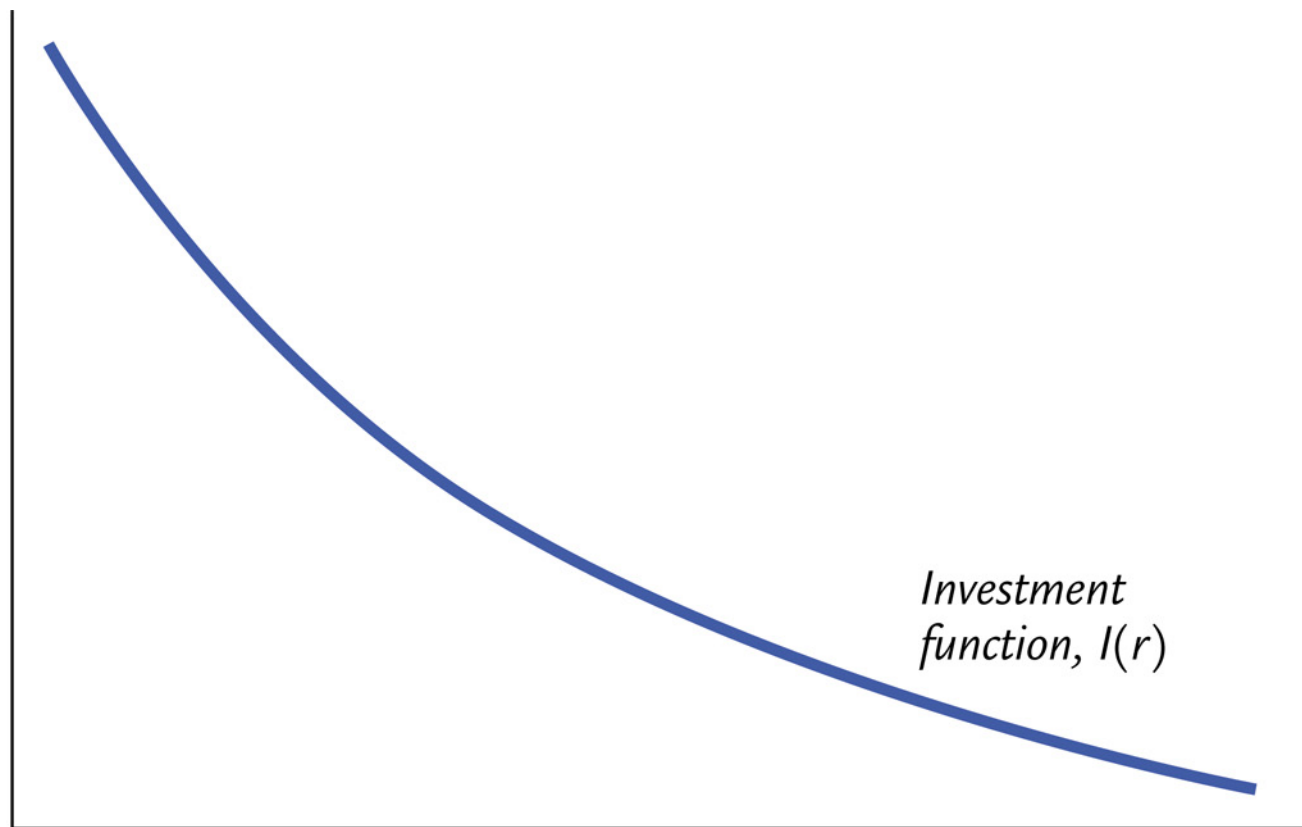
where  $r$  is the real interest rate,  $i$  is the nominal interest rate, and  $\pi$  is the rate of inflation.

Investment demand function:

$$I = I(r)$$

Investment demand curve is downward sloping!

Real interest rate,  $r$



*Investment  
function,  $I(r)$*

Quantity of investment,  $I$

# GOVERNMENT EXPENDITURES $\approx 22\%$ OF GDP

We assume that both government expenditures,  $G$  and aggregate taxes net of aggregate transfers (e.g., unemployment insurance payments, medical insurance, etc.),  $T$  are fixed.

$$G = \bar{G}$$

$$T = \bar{T}$$

If  $G - T > 0$ , then government runs a budget deficit;

If  $G - T < 0$ , then government runs a budget surplus;

If  $G = T$ , then a budget is balanced.

# BRINGING SUPPLY AND DEMAND TOGETHER

What is the mechanism that ensures equality of aggregate supply and demand? It is the real interest rate.

Taking all pieces of aggregate demand together:

$$Y = C + I + G = C(Y - T) + \bar{G} + I(r),$$
$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}.$$

Note that everything is fixed in the equation but the investment demand. Thus, the interest rate adjusts to bring the fixed aggregate supply into equilibrium with an aggregate demand—through the investment demand.

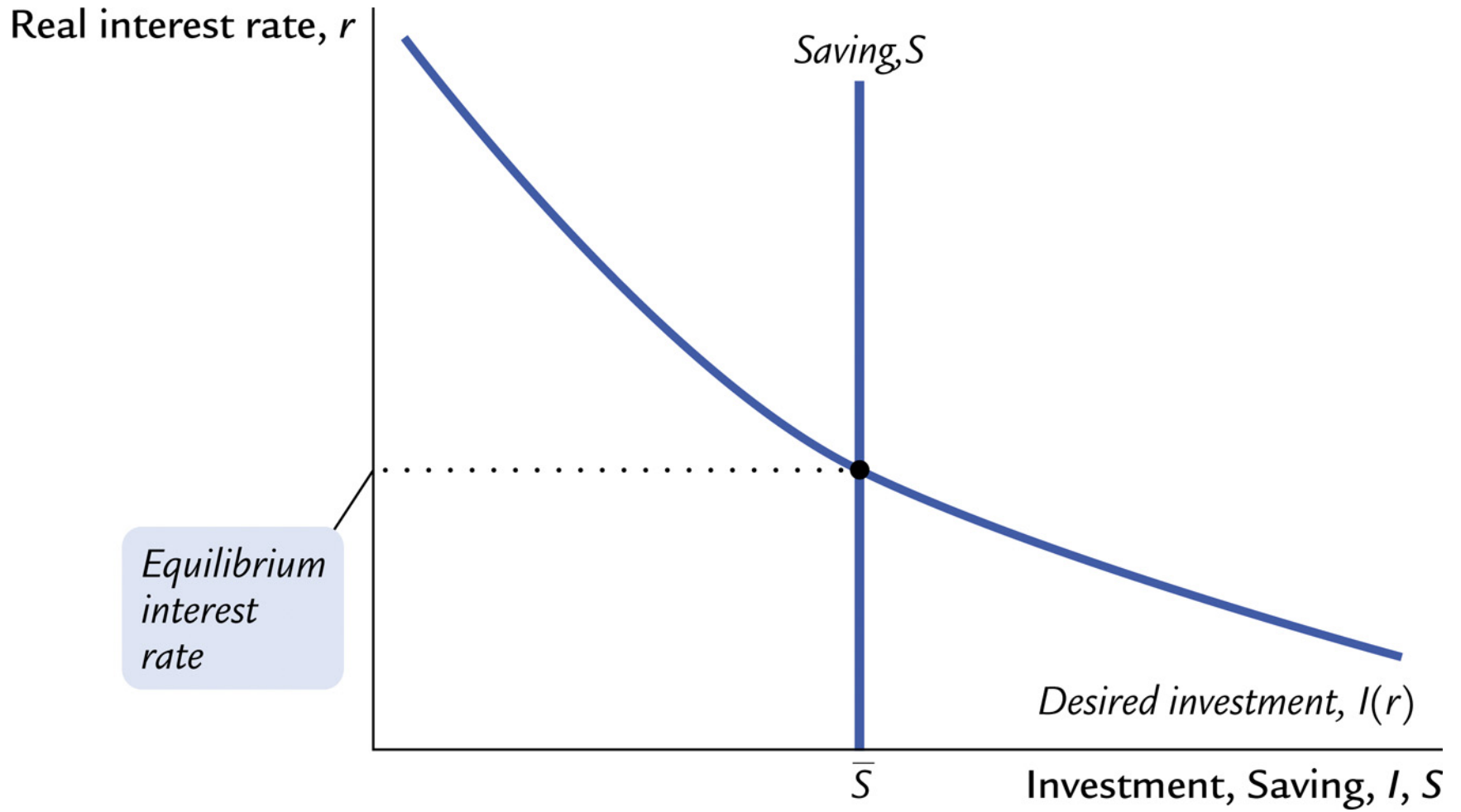
# FINANCIAL MARKETS

Rearranging the expression above,

$$\bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G} = I(r)$$

$$\underbrace{(\bar{Y} - C(\bar{Y} - \bar{T}) - \bar{T})}_{\text{private aggregate savings}} + \overbrace{(\bar{T} - \bar{G})}^{\text{public (government) savings}} = I(r)$$

where  $\underbrace{\hspace{2cm}}$  is the private aggregate savings, and  $\overbrace{\hspace{2cm}}$  is the public (government) savings.



## Effect of Exogenous Changes in $G$ and $T$

Assume  $G$  increases by the amount  $\Delta G$ . What happens in the economy?

- Simple answer: since taxes are unchanged,  $I$  should fall by the same amount to satisfy the national accounts identity.  $r$  should increase to accommodate the fall in  $I$ .
- More sophisticated answer:  $G \uparrow \rightarrow$  Public Savings  $\downarrow \rightarrow$  Shift in Savings curve to the Left  $\rightarrow r \uparrow \rightarrow I \downarrow$

It is usually said that an increase in government purchases crowds out private investment: we observe the same income and output in the economy, but the composition of aggregate demand is now different, with more  $G$  and less  $I$ .

Practice: What happens to the components of aggregate demand if  $T$  decreases by the amount  $\Delta T$ ? increases by the amount  $\Delta T$ ?

Practice: Problems 1-7, 9.



Real interest rate,  $r$

