

# CHAPTER 16: MICRO-FOUNDATIONS: CONSUMPTION

Instructor: Dmytro Hryshko

# WHY STUDY CONSUMPTION?

- Consumption is about 2/3 of GDP.
- Is our consumption function  $C = C(Y - T)$  too simplistic? Importantly, is it realistic?

# J. M. KEYNES'S CONJECTURES ABOUT THE CONSUMPTION FUNCTION

- 1  $0 \leq MPC \leq 1$ : out of each additional dollar, we spend  $MPC$  and save  $1 - MPC$  dollars.
- 2 The average propensity to consume,  $APC = \frac{C}{Y}$ , falls as income increases. I.e., richer people save a higher proportion of their incomes.
- 3 Consumption is irresponsive to the real interest rate.

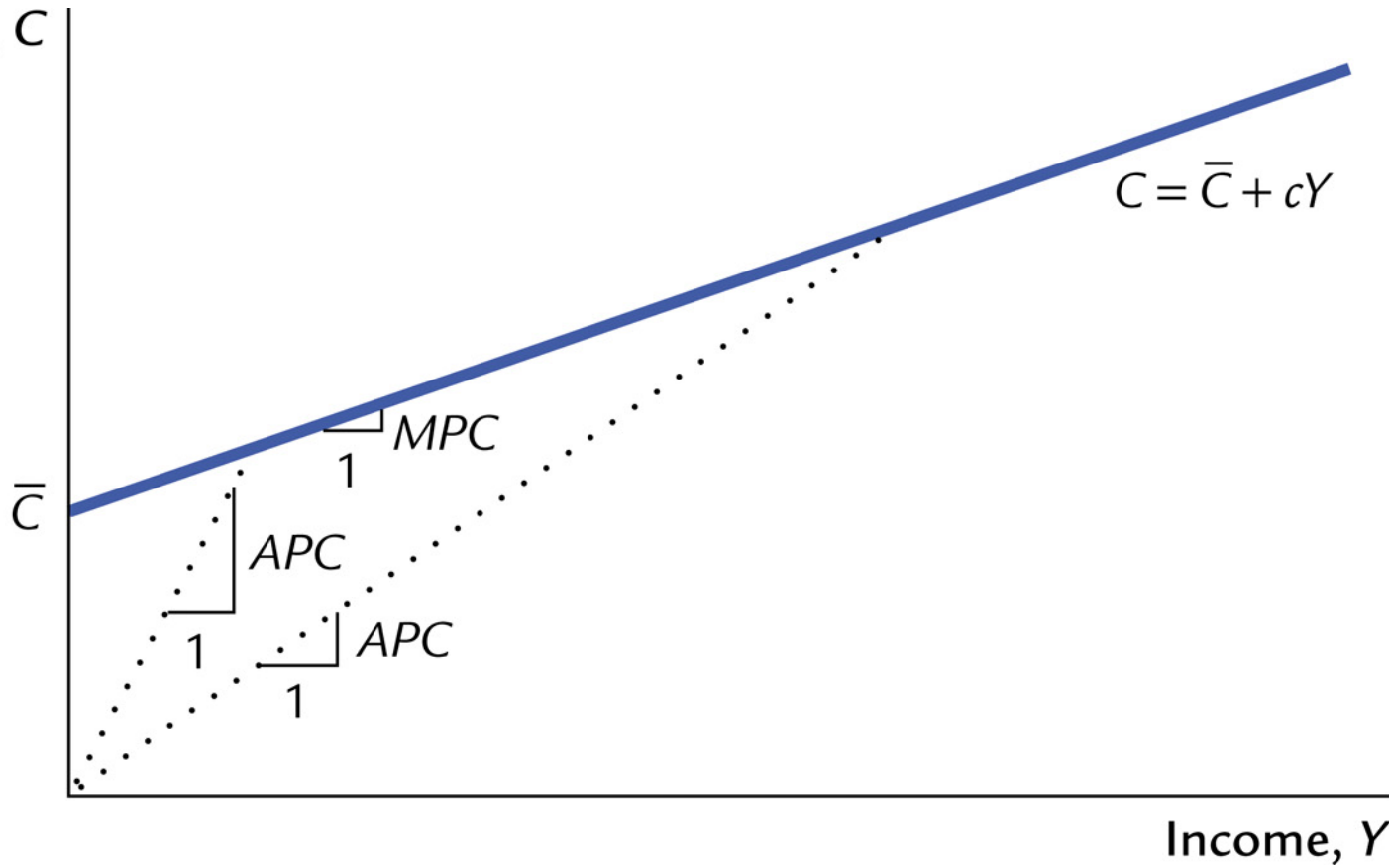
# THE KEYNESIAN CONSUMPTION FUNCTION

Summarizing, the Keynesian consumption function can be written as:

$$C = \bar{C} + c \times Y, \quad \bar{C} > 0, \quad 0 < c < 1.$$

$$APC = \frac{C}{Y} = \frac{\bar{C}}{Y} + c.$$

Consumption,  $C$



# SUCCESSSES AND FAILURES

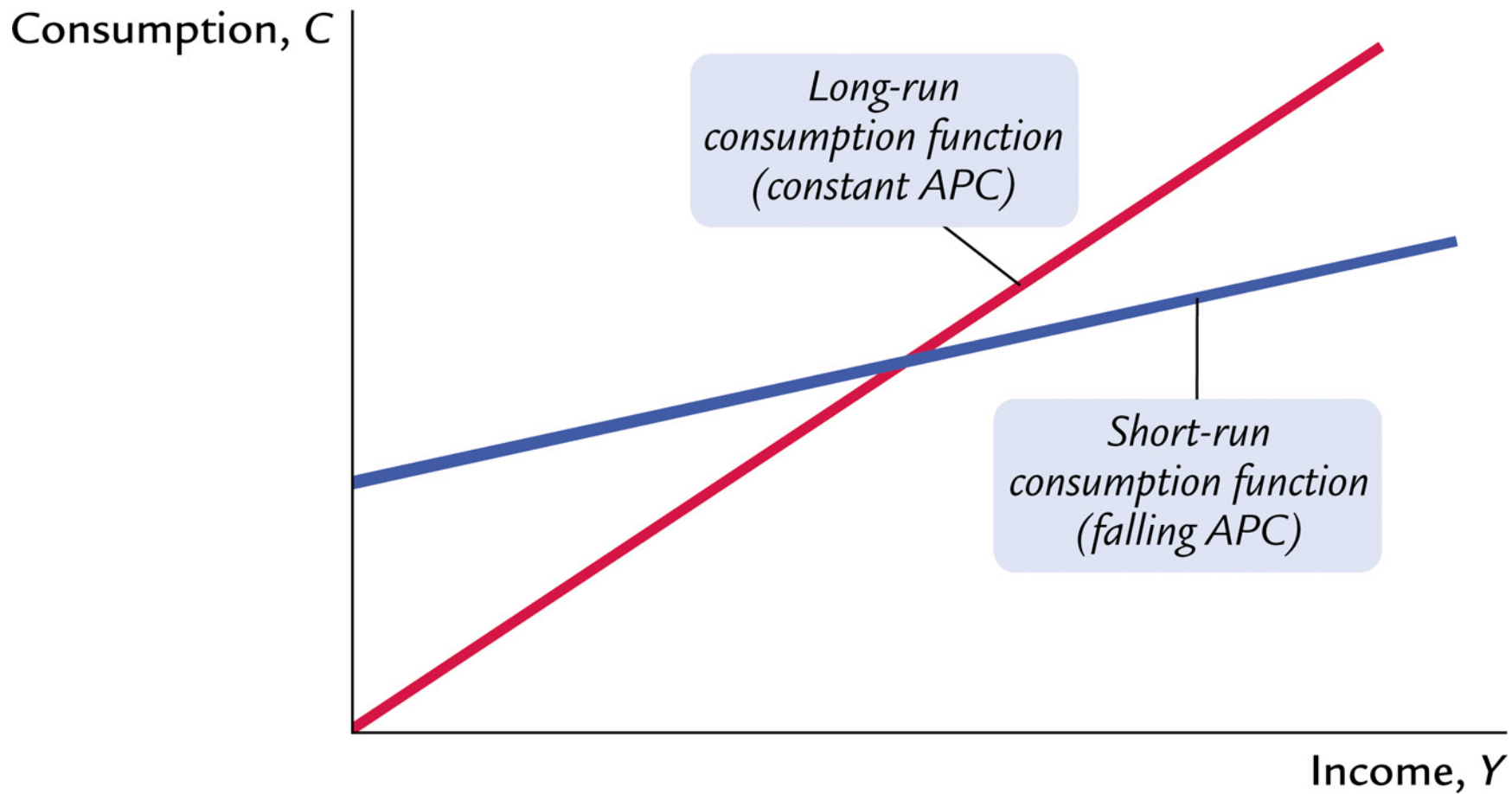
## Successes:

- Household data:  $0 < MPC < 1$ ,  $APC$  is smaller for higher income households.
- Aggregate data (in-between the wars, when income was low): the ratio of  $C$  to  $Y$  was high;  $Y$  was the primary determinant of  $C$ .

## Failures:

- Falling  $APC$  + rising incomes during the *WWII* would lead to a *secular stagnation*—a long depression in absence of changes in  $G$  or  $T$ . This prediction about falling  $APC$  did not hold.
- S. Kuznets assembled consumption and income data back to 1869.  $\frac{C}{Y}$ , i.e. the  $APC$  was stable.

*Reconciliation* of successes and failures: two consumption functions—for the short- and the long-runs.



# I. FISHER AND THE INTERTEMPORAL CHOICE

- In reality consumption responds not only to changes in *current* income but also to changes in (*expected*) income from future periods.

Let consumer live for 2 periods: period 1—youth/adulthood, period 2—old age. Let  $S_1$  be savings in period 1;  $C_j$  and  $Y_j$  are consumption and income in period  $j$  ( $j=1,2$ );  $r$ —the real interest rate.

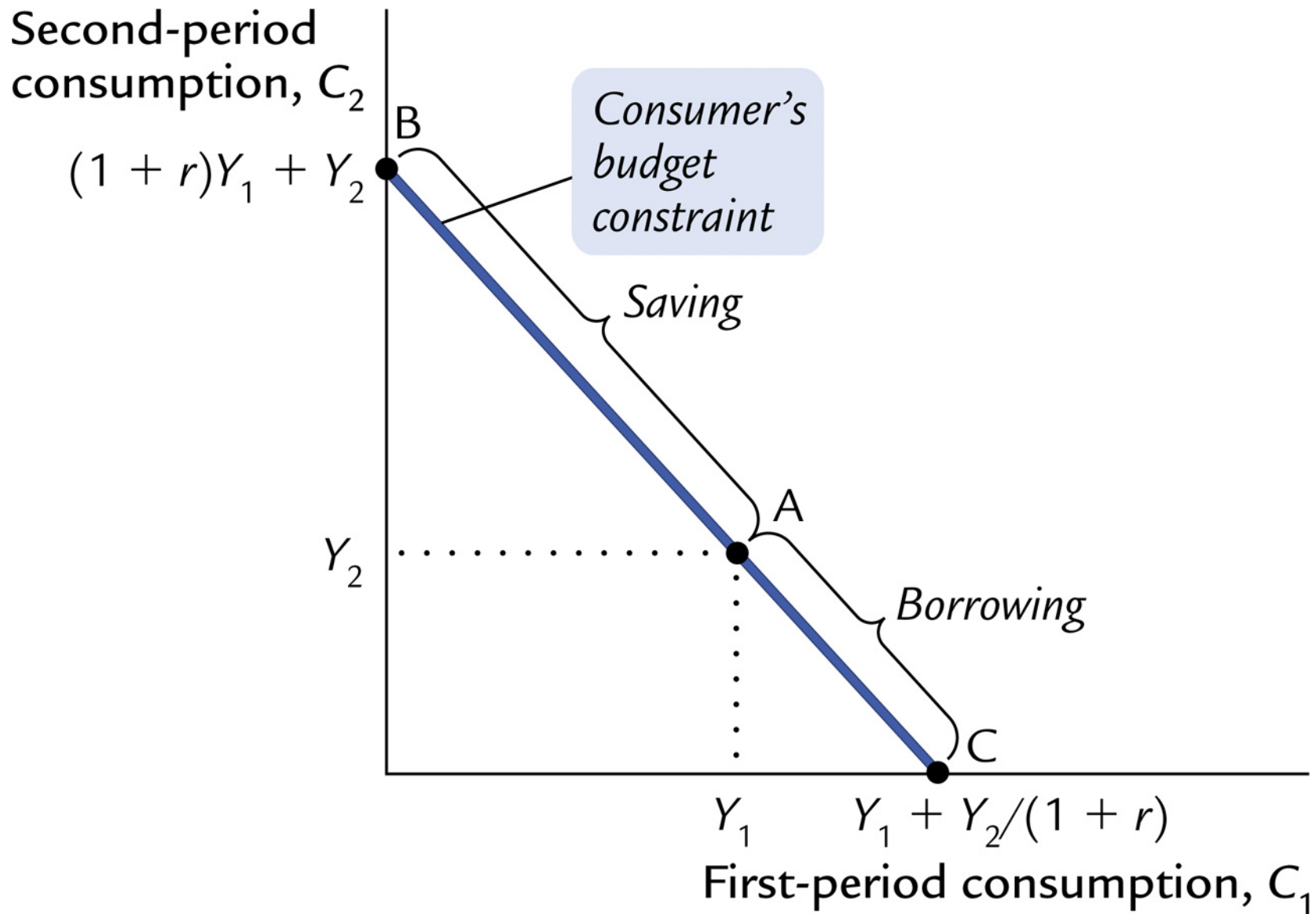
Then,  $S_1 = Y_1 - C_1$ ,  $C_2 = S_1(1 + r) + Y_2$ . Plugging  $S_1$  into the second equation, we obtain the intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}.$$



# OPTIMIZATION

If the preferences are represented by utility function  $U = U(C_1, C_2)$ , then the consumer chooses  $C_1$  and  $C_2$  that bring the highest utility index such that the budget constraint is exhausted.



## OPTIMIZATION CONTD. (1)

- The *indifference curve*: any combination of  $C_1$  and  $C_2$  that bring the same utility index  $U$ . The slope of indifference curve is defined from:  $dU = MU_1 dC_1 + MU_2 dC_2$ , or  $0 = MU_1 dC_1 + MU_2 dC_2$ , i.e.,  $\frac{dC_2}{dC_1} = -\frac{MU_1}{MU_2}$ .
- The slope of the budget constraint:  $-(1+r)$ .
- When consumer maximizes utility, *on the margin*, the benefit of adjusting his optimal bundle of  $(C_1^*, C_2^*)$  should be zero.  
*The period-1 cost* of reducing consumption by  $dC_1$  is  $MU_1 \times dC_1$ , and *the period-2 benefit* of this reduction is  $MU_2 \times dC_1 \times (1+r)$ .

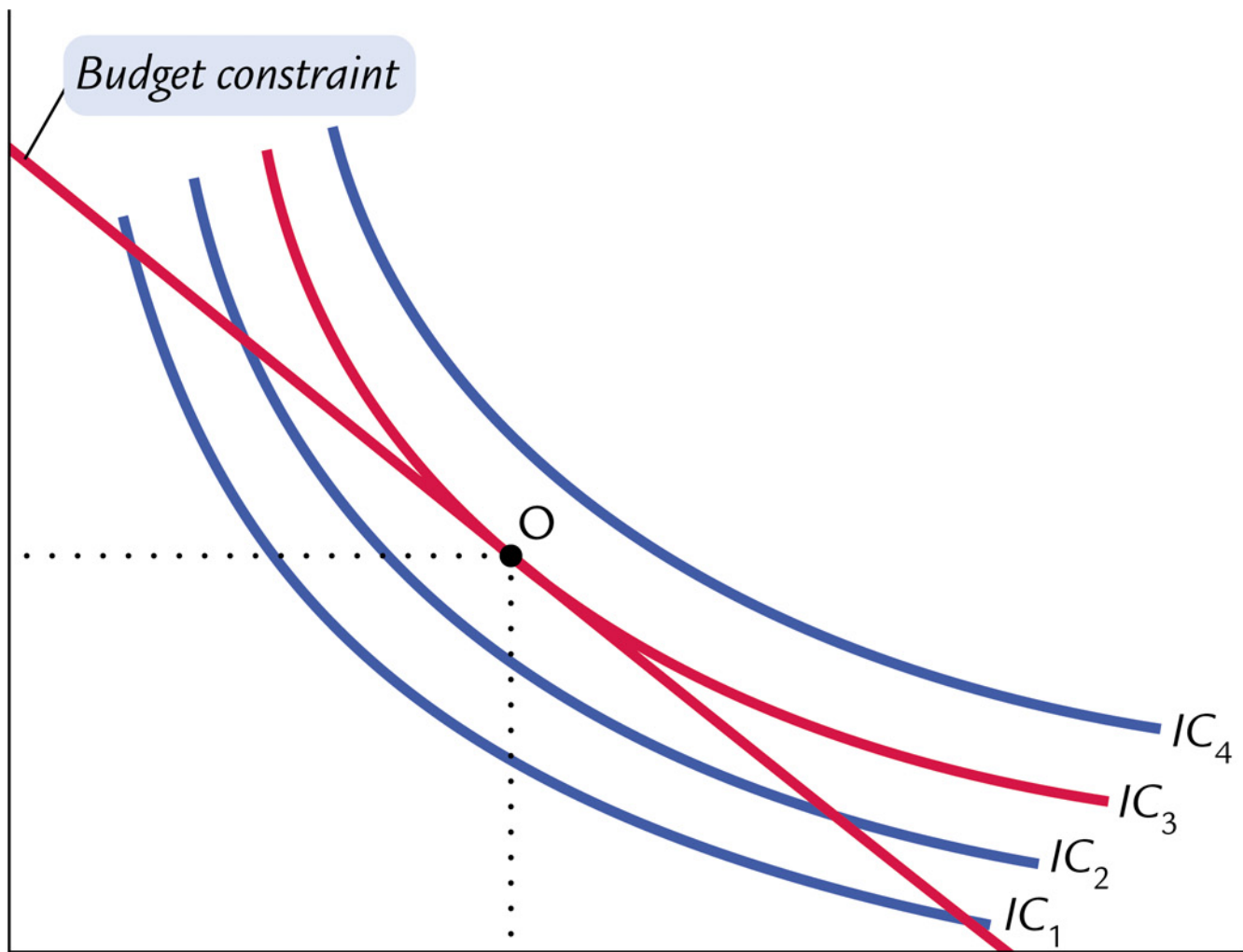
Thus, at the optimum:  $MU_1 \times dC_1 = MU_2 \times dC_1 \times (1+r)$ , or

$$-\frac{MU_1}{MU_2} = -(1+r).$$

## OPTIMIZATION CONTD. (2)

I.e., the highest possible utility is achieved at the point  $(C_1^*, C_2^*)$ , where *the slope of the indifference curve* is equal to *the slope of the budget constraint*.

Second-period consumption,  $C_2$



*Budget constraint*

O

$IC_4$

$IC_3$

$IC_2$

$IC_1$

First-period consumption,  $C_1$

## FROM THE CONSTRAINT IN NOMINAL TERMS TO THE CONSTRAINT IN REAL TERMS

In nominal terms, the budget constraint can be derived going through the following steps. If you save \$1 of income in period 1, you will be entitled to  $\$(1 + i)$  in period 2. Denote the price of 1 unit of consumption in period 1 as  $P_1$ ,  $P_2$  is the same for period 2.

Your endowments of goods 1 and 2 are equal to  $Y_1$  and  $Y_2$ , respectively. Their nominal value is  $P_1 C_1$  and  $P_2 C_2$ .

Savings in the first period in nominal terms are  $S_1 = P_1(Y_1 - C_1)$ . Next-period nominal resources are  $(1 + i)P_1(Y_1 - C_1) + P_2 Y_2$  and  $P_2 C_2 = (1 + i)P_1(Y_1 - C_1) + P_2 Y_2$ . Divide the RHS and the LHS of the equation by  $P_2$  to obtain,

$$\begin{aligned} C_2 &= (1 + i) \frac{P_1}{P_2} (Y_1 - C_1) + Y_2 \\ &= \frac{1 + i}{1 + \pi} (Y_1 - C_1) + Y_2 = (1 + r)(Y_1 - C_1) + Y_2. \end{aligned}$$

# INCOME AND SUBSTITUTIONS EFFECTS (1)

This boils down to the lifetime constraint in **real** terms:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}.$$

Compare it to the constraint for static optimization with two distinct goods and income,  $I$ :

$$P_X X + P_Y Y = I.$$

In terms of this notation,  $1 = P_X$ ,  $\frac{1}{1+r} = P_Y$ , and  $Y_1 + \frac{Y_2}{1+r} = I$ . In our dynamic constraint, the price of  $C_1$  is normalized to 1, and the price of  $C_2$  in terms of  $C_1$  is  $\frac{1}{1+r}$ .

If  $r$  increases,  $C_2$  becomes cheaper in terms of  $C_1$ .

## INCOME AND SUBSTITUTION EFFECTS (2)

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}.$$

If  $Y_1$  or  $Y_2$  changes, this impacts only the lifetime income. Hence, these changes will cause changes in  $(C_1^*, C_2^*)$  due to the *income effect*.

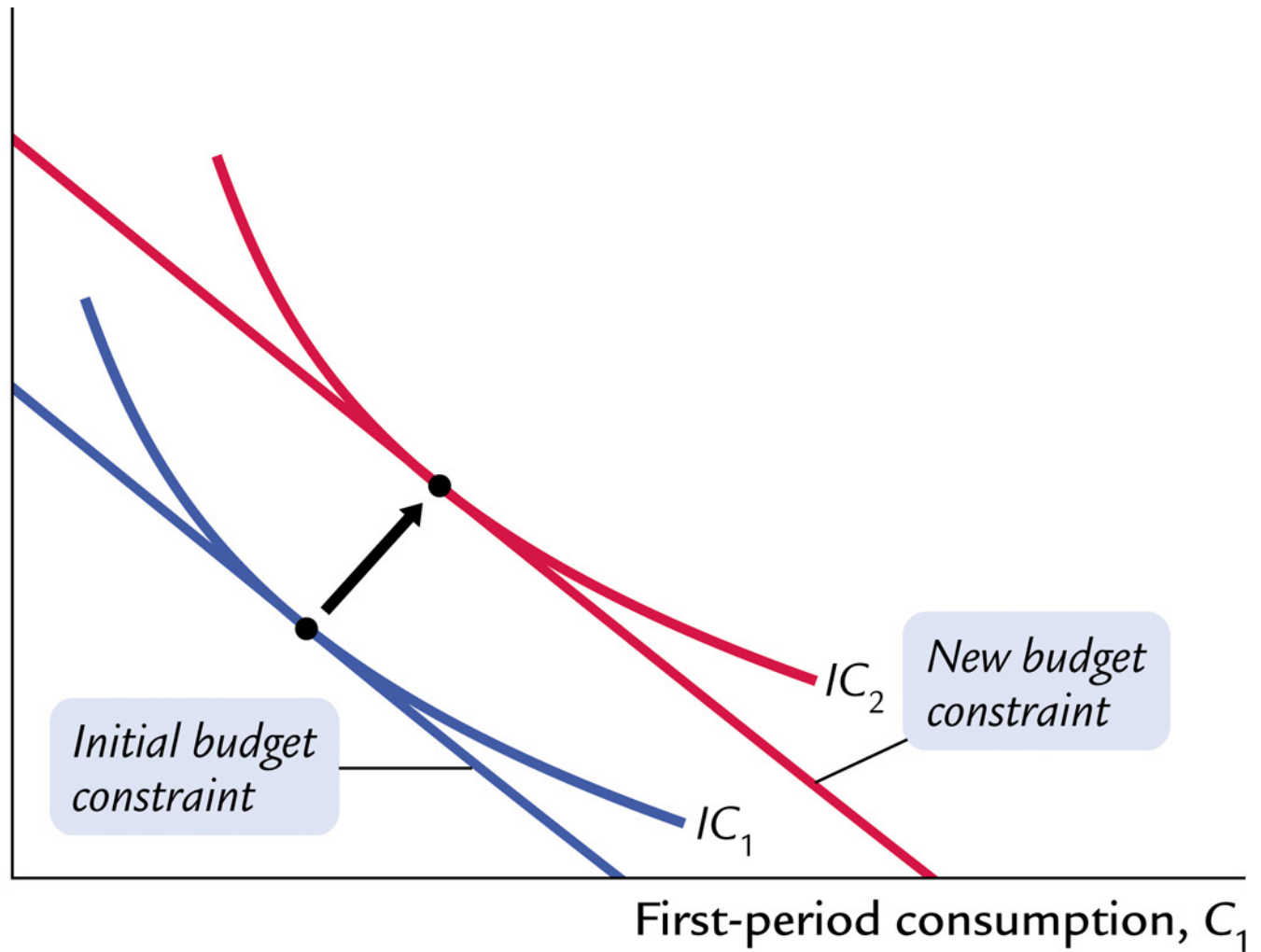
If  $r$  changes, it impacts both the lifetime income *and* the relative price of consumption in periods 1 and 2. Thus, the *substitution effect* will be more complicated and will be different for savers ( $C_1^* < Y_1$ ) versus borrowers ( $C_1^* > Y_1$ ).



## INCOME AND SUBSTITUTION EFFECTS (3)

- If  $Y_1$  or  $Y_2$  increases, both  $C_1$  and  $C_2$  increase, provided they are *normal* goods.
- Thus, the timing of income is irrelevant for current consumption decisions if the consumer is not constrained in borrowing.
- Effects on  $C_1^*$  and  $C_2^*$ , following a change in the real interest rate, depend on whether the consumer is a saver ( $C_1 < Y_1$ ), or a borrower, ( $C_1 > Y_1$ ).
- If the consumer is a *saver* and  $r \uparrow$ ,  $C_1$  should *increase* because of the income effect, and *decrease* because of the substitution effect (since the price of the period-2 consumption is now lower). The final change in  $C_1$  is ambiguous;  $C_2$ , though, increases unambiguously.

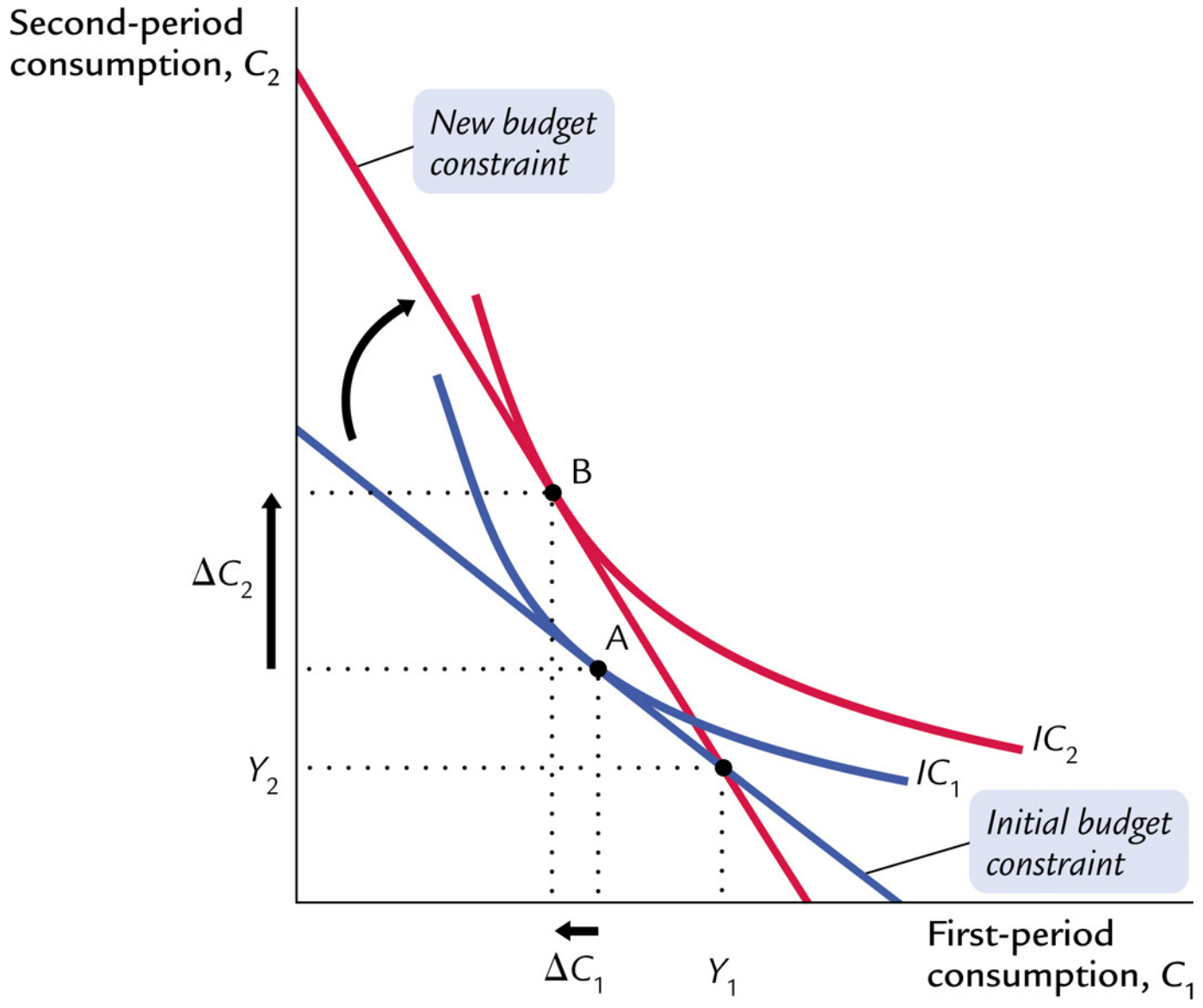
Second-period consumption,  $C_2$



*Initial budget constraint*

*New budget constraint*

First-period consumption,  $C_1$

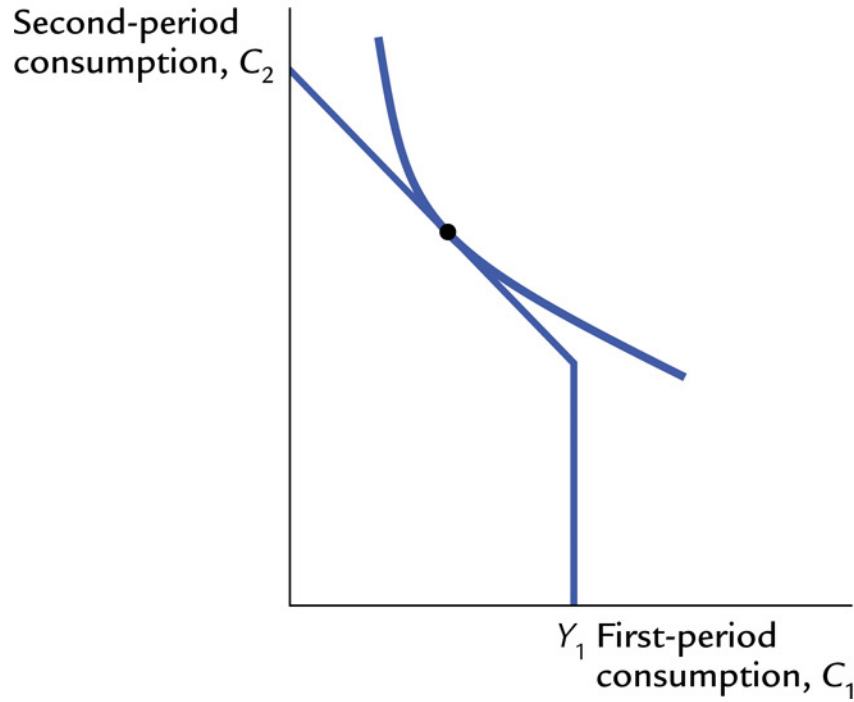


# LIQUIDITY CONSTRAINTS

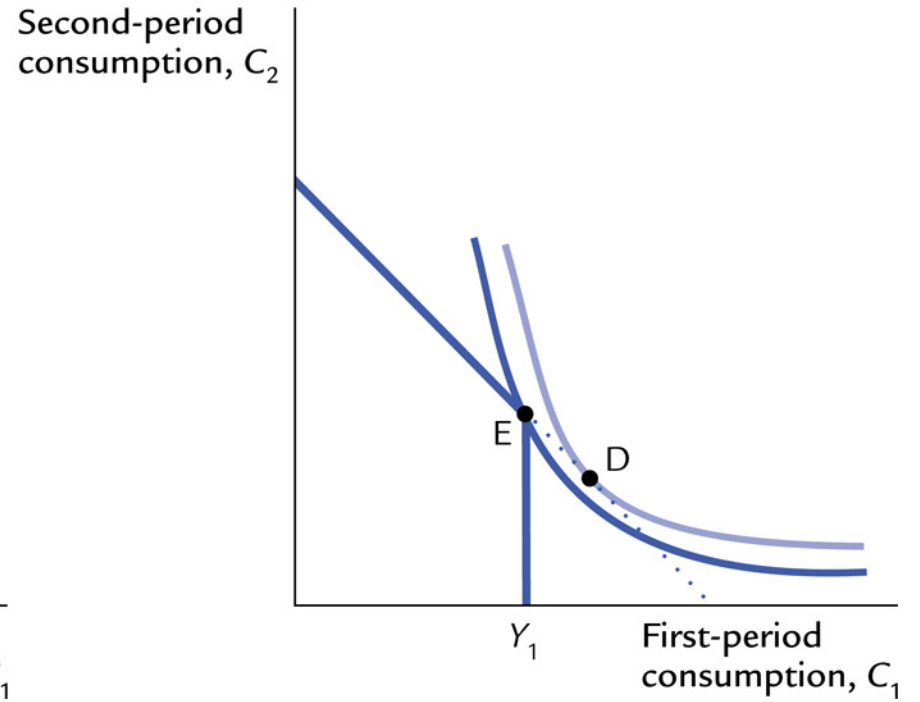
If the consumer is a would-be-'borrower' but cannot borrow, i.e., is liquidity constrained, then this consumer's  $C_1$  will be equal to  $Y_1$ .

Thus, for liquidity constrained consumers current consumption is determined by current income.

**(a) The Borrowing Constraint Is Not Binding**



**(b) The Borrowing Constraint Is Binding**



Consider first the problem without liquidity constraints. Then,  $(C_1^*, C_2^*)$  should satisfy 2 equations. The Euler equation:

$$MU_1(C_1^*) = (1 + r)MU_2(C_2^*),$$

and the budget constraint:

$$C_1^* + \frac{C_2^*}{1 + r} = Y_1 + \frac{Y_2}{1 + r}.$$

We're assuming that the marginal utility in period 1 is a function of consumption in period 1 only. Similarly, for period 2.

With liquidity (borrowing) constraints,  $C_1$  cannot be larger than  $Y_1$ , that is,  $C_1 \leq Y_1$ . If the unconstrained problem gives you  $C_1^* \leq Y_1$ , then we say the constraint is *not binding*—consumer is a saver anyways. Otherwise, if  $C_1^* > Y_1$ , the maximum this consumer can have in period 1 is  $Y_1$ . Summarizing,  $C_{1,bc}^* = \min\{C_1^*, Y_1\}$ ;  $C_{2,bc}^* = C_2^*$  if  $C_{1,bc}^* = C_1^*$ ,  $C_{2,bc}^* = Y_2$  otherwise.

## Example

Let the utility function be  $U(C_1, C_2) = \log(C_1) + \beta \log(C_2)$ , where  $\log$  is the natural logarithm. Let  $Y_1 = 40$ ,  $Y_2 = 80$ ,  $r = 0.05$ ,  $\beta = 0.90$ . First, assume that consumer is unconstrained.

For this utility function,  $MU_1 = \frac{1}{C_1}$ , and  $MU_2 = \beta \frac{1}{C_2}$ . The Euler equation tells us that  $\frac{1}{C_1} = \beta(1+r)\frac{1}{C_2}$ , or  $C_2^* = \beta(1+r)C_1^*$ . Plug this result into the budget constraint to obtain,

$$C_1 + \frac{\beta(1+r)C_1^*}{1+r} = Y_1 + \frac{Y_2}{1+r}, \text{ or}$$

$$C_1^* = \frac{1}{1+\beta} \left[ Y_1 + \frac{Y_2}{1+r} \right] = \frac{1}{1.90} (40 + 80/(1.05)) \approx 61. \text{ Thus,}$$

$$C_2^* = 0.90 * (1.05) * 61 \approx 57.8.$$

If consumer is constrained this implies that  $C_{1,bc}^* < Y_1$ , and  $C_{1,bc}^* = \min\{61, 40\} = 40$ . Also,  $C_{2,bc}^* = 80$ . You can also show that the liquidity constrained consumer will be worse off.

# THE LIFE-CYCLE HYPOTHESIS

- Idea: want to smooth consumption over the life cycle. Thus, need to save during the working life to support consumption during retirement.

Assume consumer prefers a smooth consumption path; will live  $T$  more years; will work for another  $R$  years; has wealth of  $W$ ; and will receive a sure income  $Y$  during the working years.

The lifetime wealth is  $W + R \times Y$ . Then,

$$C = (W + R \times Y)/T = W/T + (R/T) \times Y = \alpha \times W + \beta \times Y.$$

Thus,  $\frac{C}{Y} = \alpha \times (W/Y) + \beta$ .

In the short run, for a fixed  $W$ , an increases in  $Y$  leads to a falling  $APC$ ; in the long run,  $W$  and  $Y$  grow together, and  $C/Y$  is stable.

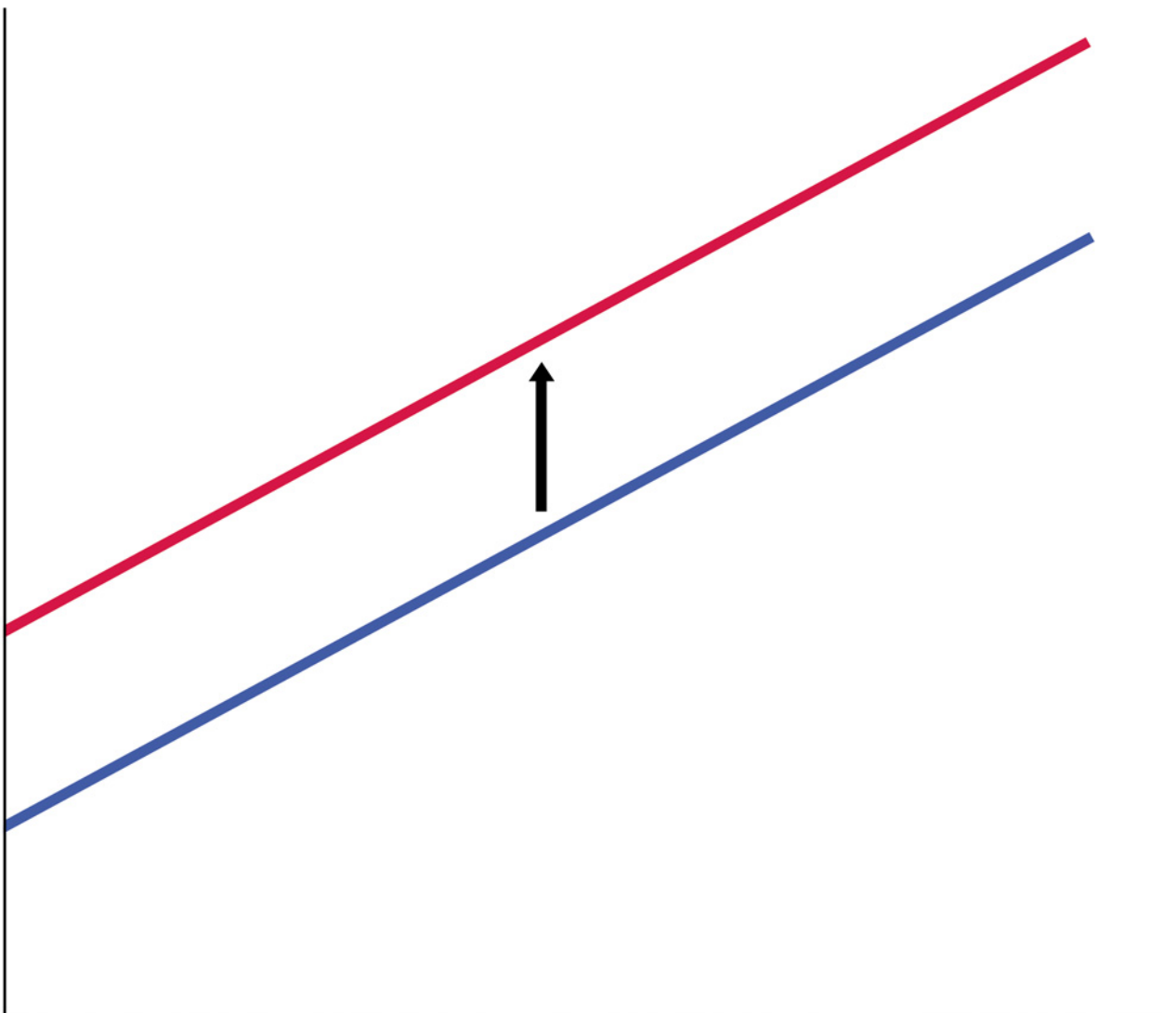


Consumption,  $C$

$\alpha W_2$

$\alpha W_1$

Income,  $Y$



# MILTON FRIEDMAN'S PERMANENT INCOME HYPOTHESIS

Milton Friedman postulated the following model of consumption:

$$\text{Income : } Y = Y^T + Y^P,$$

$$\text{Consumption : } C = Y^P.$$

$Y^P$  is *the permanent income* (the component of income that persists over time);  $Y^T$  is *the transitory income* (short-lived components of income such as bonuses, overtime, windfalls from lottery etc.)

# THE PERMANENT INCOME HYPOTHESIS (PIH)

## Implications:

- Consumption changes by the magnitude of a change in *permanent income*. Transitory changes in income are predominantly saved.
- $APC = \frac{C}{Y} = \frac{Y^P}{Y}$ . In the household studies, most of variation in  $\frac{Y^P}{Y}$  comes from the transitory variation in income. Thus, if  $Y > Y^P$ ,  $C < Y$ , and the  $APC$  is falling.
- In the long time series, most of variation in  $Y$  comes from the variation in  $Y^P$ , and so  $APC$  will be stable.

# HALL'S FORMULATION OF THE PIH

*PIH*: consumption depends on  $Y^P$ , and therefore on expectations of the lifetime resources.

*Robert Hall*: if the *PIH* holds, and consumers form rational expectations, then *changes in consumption are unpredictable*. I.e., (the level of) consumption is a *martingale*, and consumption responds only to the 'news' in income.

Importantly, under the *PIH* only *unexpected* changes in policy influence consumption.