Intermediate Macroeconomic Theory II, Fall 2014.<br>Instructor: Dmytro Hryshko<br>Problem set 120 (points). Due November 26, by 4 PM.

1. ( $\mathbf{3 0}$ points) Consider a variant of the "lake" model of the natural rate of unemployment we studied in class. Assume the labor force is constant and equals $L$. At any point in time, labor force is split into three groups ("lakes")-long-term employed, $E_{l}$, shortterm employed, $E_{s}$, and unemployed, $U$. The flows between the lakes are the following: unemployed workers join the pool of short-term employed at a rate $f$ but never join the pool of long-term employed directly; short-term employed join the pool of long-term employed at a rate $t$ and become unemployed at a rate $s$, while long-term employed never join the pool of short-term employed directly but become unemployed at a rate $\mu$. (Hint: you may find it useful to visualize the problem by drawing the "lakes" and the flows between them.)
(a) Derive the steady-state natural rate of unemployment as a function of the model parameters, $f, \mu, s$, and $t$.
(b) Under what condition will the natural rate of unemployment be zero? Provide the intuition. Calculate the natural rate of unemployment when $\mu=0.005, s=0.01$, $t=0.07$, and $f=0.2$.
(c) What happens to the natural rate of unemployment when $t$ increases (assume that $s>\mu)$ ?
2. ( 30 points) Consider the DMP model of frictional unemployment we studied in class. Assume that legislators are contemplating about giving a subsidy $s$ to any firm that hires a worker in order to reduce the economy's unemployment rate. Explain whether the policy would be successful using the model, and draw the relevant diagrams. (Hint: note that the firm's surplus will be modified to $(z+s-w)$, while the total surplus to $(z+s-b)$; the rest of the analysis is the same.)
3. (30 points) Consider a variant of the 2-period model of credit markets with limited commitment we studied in class (all of the model parameters can be taken from the slides). Suppose there's limited commitment in the credit market with lenders being uncertain about the value of collateral. Each consumer has a quantity of collateral $H$ but from the point of view of lender, there is a probability $a$ that the collateral will be worth $p$ in the future period, and probability $1-a$ that the collateral will be worthless in the future period. Suppose that all consumers are identical.
(a) Determine the collateral constraint for the consumer, and show the consumer's lifetime budget constraint in a diagram (be sure to show the consumer's endowments in the current and future periods on your graph).
(b) How will a decrease in $a$ affect the consumer's consumption and savings in the current period, and consumption in the future period? Explain your results.
4. (30 points) Consider a Central Bank that maximizes the following utility function:

$$
U=k\left(y-y_{e}\right)-\left(\pi-\pi^{T}\right)^{2},
$$

where $k$ is a positive constant, $y_{e}$ is the full-employment level of output. Its policy instrument is the growth rate of the money supply, $\gamma_{M}$. Assume that the inflation target is $\pi^{T}=0$. Assume that the central bank sets the money supply growth rate after economic agents have incorporated their expectations about inflation into their decision making, and thus faces a Phillips curve:
$\pi=\pi^{e}+\alpha\left(y-y_{e}\right)$.
(a) Explain the utility function and compare it with the loss function used in the class. (Hint: focus on how the central bank's utility rises with output. Is this central bank 'overambitious'?)
(b) Assuming that agents have rational expectations, solve algebraically for the optimal inflation rate under discretion, i.e., find the inflation rate that the central bank will choose using its monetary policy instrument, $\gamma_{M}$. (Hint: maximize utility with respect to $\gamma_{M}$, having used the Phillips curve to substitute for $y$ in the utility function; and having used $\gamma_{M}=\pi$ to substitute for $\pi$.)
(c) Suppose that, before private sector inflation expectations were formed, the central bank could commit to a particular rate of inflation. What would that rate be? Discuss.

