

Name: _____

Intermediate Macroeconomic Theory II, Fall 2014.

Instructor: Dmytro Hryshko

Problem set 1 (60 points). Due October 10, by 4 PM.

1. **(15 points)** Let the economy's production function be $Y = 5K^{1/2}(EL)^{1/2}$.
Households save 40% of their income;
population growth, n , is equal to 2%;
the depreciation rate, δ , is equal to 1%;
the growth rate in the efficiency of labor, g , is 2%.
 - (a) **(2 points)** Show that the aggregate production function is constant returns to scale in K and L **(1 point)**, and express the production function in *per-effective-worker* terms **(1 point)**.
 - (b) **(1 point)** Is production function increasing/constant/decreasing returns to scale in 3 factors of production, K , E , and L ? *Show* how you arrived at the conclusion.
 - (c) **(3 points)** Calculate the steady state level of
capital *per effective worker* **(1 point)**
output *per effective worker* **(1 point)**
and consumption *per effective worker* **(1 point)**.
 - (d) **(2 points)** Find the ratio of capital per worker to output per worker in the economy on a balanced growth path, and the real rental price of capital.
 - (e) **(1 point)** What is the growth rate of the real wage in the economy on a balanced growth path? What is the growth rate of the total output?
 - (f) **(1 point)** If you were a social planner who maximizes consumption per worker in the economy, what savings rate would you choose? (You *need not* show your calculations here if you see the answer.)
 - (g) **(3 points)** Find the golden rule level of capital per effective worker **(1 point)** and the corresponding output per effective worker in the golden rule steady state **(1 point)**, and consumption per effective worker **(1 point)**.
 - (h) **(2 points)** Assume the government can tax proportionally capital and wage incomes at the rate τ , and invests only the fraction s (equal to the private savings rate) of the tax proceedings, while wasting the rest. What is the level of consumption per effective worker in the very long run under this tax policy, assuming that s equals 40%? (*Hint*: it will be a function of the tax rate.) How does it compare to consumption per effective worker in (1c)?

2. **(15 points)** Let us modify the Solow growth model without technological progress by including government spending as follows. The government purchases G units of consumption goods in the current period where $G = gL$, and g is a positive constant. The government finances its purchases through lump-sum taxes on consumers, where T denotes total taxes, and the government budget is balanced each period, so that $G = T$. Consumers consume a constant fraction of disposable income, that is, $C = (1 - s)(Y - T)$, where s is the savings rate, saving the rest. Assume that capital depreciates at the rate δ , labor force grows at the rate n , and production function is constant-returns to scale in K and L , $Y = F(K, L)$.
- (5 points)** Derive the fundamental equation for the law of motion of capital per worker.
 - (5 points)** Show graphically that there can be two steady states, one with high capital per worker, k^* , and one with low k^* .
 - (5 points)** Ignore the steady state with low k^* . (It can be shown that this steady state is “unstable.”) Determine the effects of an increase in g on capital per worker and output per worker in the steady state. What are the effects on the growth rates of aggregate output, aggregate consumption, and aggregate investment? Thoroughly explain your results.
3. **(15 points)** Consider the Permanent Income Hypothesis we studied in class. Preferences are quadratic, $u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$; planning horizon is infinite and consumer starts her life at time 0; $\beta(1 + r) = 1$; income stream is known as of time 0. Consider individual X whose income is 0 in even periods (periods 0, 2, 4, 6, etc.) and 190 in odd periods (periods 1, 3, 5, 7, etc.); and individual Y whose income equals 100 in all periods. The real interest rate, r , equals $1/9$, and $\frac{1}{1+r} = 0.9$.
- (5 points)** Find the present discounted value of incomes for individual X and Y . Who is lifetime richer?
 - (5 points)** Find the optimal consumption plan for both individuals assuming they are not borrowing constrained.
 - (5 points)** Qualitatively describe the optimal consumption plan for both individuals if they are not allowed to borrow at all (there’s no need for calculations here but you may provide exact numbers if you choose so).
4. Briefly comment on the following statements, indicating whether they are true or false where appropriate, and show how you arrived at your conclusion.
- (3 points)** The real interest rate is constant in the steady-state of the Solow economy with population growth but no technological growth, while it is growing on a balanced growth path of the economy with positive population and technological growth.
 - (3 points)** Ceteris paribus, we can predict that the price of capital and the real interest rate will go down if some part of the economy’s capital stock gets destroyed, say, due to war.

- (c) (**3 points**) Ceteris paribus, we can predict that the price of capital and the real interest rate will go down if there is an inflow of immigrants into the economy.
- (d) (**3 points**) Consider the following situation. An individual receives some news that she will receive a large inheritance in 10 years from now with a very high likelihood. The Permanent Income Hypothesis predicts that the person will not act on this news now in terms of current consumption but will rather wait for 10 years, and only then change her consumption if that inheritance is indeed received.
- (e) (**3 points**) Consider the Solow economy without technological growth, in its steady state equilibrium. At some point in time, part of the economy's stock of capital is destroyed but otherwise the economy remains the same as before, that is the savings patterns are the same, population growth, depreciation rate, as well as the aggregate technology are the same. What will happen eventually, in the very long run, to this economy in terms of capital per worker and output per worker? What will happen to the economy during its transition to that long-run state?