ECON385: A note on the Permanent Income Hypothesis (PIH).

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In this note, we will try to understand the permanent income hypothesis (PIH).

Let us consider the following two-period problem. Consumer's within-period utility function is $u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2$; consumer's incomes are y_0 and y_1 in periods 0 and 1, respectively, and are known at time 0; consumer is not endowed with any financial wealth in period 0; the net real interest rate is constant and equal to r; consumers can freely borrow or lend at this interest rate; c_t is consumption in period t (t=0, 1), and $\bar{c}$ is the "bliss" consumption so that $c_t \leq \bar{c}$.[1] In terms of our class notation for a two-period problem, consumer wants to maximize $U(c_0, c_1)$ subject to the budget constraint. We can write it as $U(c_0, c_1) = -\frac{1}{2}(c_0 - \bar{c})^2 - \beta\frac{1}{2}(c_1 - \bar{c})^2$, where $\beta \in [0, 1]$ is the so-called (one-period) time discount factor—it puts the relative weight to the utility from consumption in period 1 to the consumer's overall satisfaction index, $U(c_0, c_1)$. If $\beta = 0$, the consumer discounts future entirely and does not care about consumption in period 1; if $\beta = 1$, the utility from consumption in period 1 has the same weight as the utility from consumption in period 0. Thus, the consumer will solve the following problem:

$$\max_{c_0 \geq 0, c_1 \geq 0} U(c_0, c_1) = -\frac{1}{2}(c_0 - \bar{c})^2 - \beta\frac{1}{2}(c_1 - \bar{c})^2 \tag{1}$$

$$s.t.\ \ c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r}. \tag{2}$$

Assume that $\beta = \frac{1}{1+r}$. We can solve for optimal levels of c_0 and c_1 using these two

[1] Note that for all c_t strictly less than $\bar{c}$ the utility index will assume a negative number. For $c_t = \bar{c}$, the utility index attains its maximum, equal to zero. Hence, $\bar{c}$ is called the "bliss" consumption. For this utility function, marginal utility, measured by the first derivative of utility function with respect to c_t, is equal to $-(c_t - \bar{c}) = \bar{c} - c_t \geq 0$—a property we require from most utility functions.

equations:

$$(\overline{c} - c_0^*) = \beta(1+r)(\overline{c} - c_1^*) \tag{3}$$

$$c_0^* + \frac{c_1^*}{1+r} = y_0 + \frac{y_1}{1+r}. \tag{4}$$

Since we assumed that $\beta = \frac{1}{1+r}$, we can write the first of those equations as $\overline{c} - c_0^* = \overline{c} - c_1^*$, or $c_0^* = c_1^*$. Plugging this equilibrium condition into the second equation, we obtain $c_0^* + \frac{c_0^*}{1+r} = y_0 + \frac{y_1}{1+r}$, or $c_0^* = c_1^* = \frac{1+r}{2+r}(y_0 + \frac{y_1}{1+r})$. Thus, the optimal consumption will be equal to a constant fraction of the lifetime resources, $y_0 + \frac{y_1}{1+r}$, and consumer, for these preferences, will prefer to smooth consumption across the periods perfectly.

What if a consumer has more than two periods; for example, his effective horizon is equal to infinity? We will now assume that a consumer faces an infinite endowment stream $\{y_0, y_1, y_2, \ldots\}$, each endowment known at time 0 (i.e., known in advance), and chooses an infinite consumption stream $\{c_0, c_1, c_2, \ldots\}$ optimally.[2]

In this case, a consumer will solve the following problem:

$$\begin{aligned}
&\max_{c_0 \geq 0, c_1 \geq 0, c_2 \geq 0, \ldots} U(c_0, c_1, c_2, \ldots) = -\frac{1}{2}(c_0 - \overline{c})^2 - \beta\frac{1}{2}(c_1 - \overline{c})^2 - \beta^2\frac{1}{2}(c_2 - \overline{c})^2 - \\
&- \beta^3\frac{1}{2}(c_3 - \overline{c})^2 - \ldots \\
s.t.\ \ & c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} + \frac{c_3}{(1+r)^3} + \ldots = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + \\
&+ \frac{y_3}{(1+r)^3} + \ldots
\end{aligned}$$

[2]We assume that a consumer's horizon is infinite since it is easier to deal with mathematically. The message will be the same if you use a finite horizon instead.

Why does our budget constraint looks like that? To see this, let us derive the budget constraint for a three-period problem. Note that $s_0 = y_0 - c_0$; $c_1 + s_1 = s_0(1+r) + y_1$; and $c_2 = (1+r)s_1 + y_2$. The second equation tells you that the available resources a consumer has in period 1, income in period 1 and gross accumulated savings in period 1, can be split between consumption and savings in period 1. The third equation tells us that a consumer eats up all the resources he ends up with in period 2. From the third equation, $s_1 = \frac{c_2}{1+r} - \frac{y_2}{1+r}$. Plug this result into the second equation, to obtain $c_1 + \frac{c_2}{1+r} - \frac{y_2}{1+r} = s_0(1+r) + y_1$, or $s_0 = \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} - \frac{y_2}{(1+r)^2} - \frac{y_1}{1+r}$. Plug this result into the first equation, to obtain $c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2}$. Similar logic applies to a multi-period problem. The intertemporal budget constraint will always read as: the present discounted value of consumption should not exceed the present discounted value of income (we assumed equality between the two in our formulation).

We can write this problem more compactly as:

$$\max_{c_0 \geq 0, c_1 \geq 0, c_2 \geq 0, \ldots} U(c_0, c_1, c_2, \ldots) = \sum_{t=0}^{\infty} \left[-\frac{1}{2} \beta^t (c_t - \bar{c})^2 \right] \tag{5}$$

$$s.t. \ \sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}. \tag{6}$$

Now, instead of finding just c_0^* and c_1^*, we will need to find the whole sequence $\{c_0^*, c_1^*, c_2^*, \ldots\}$. It turns out it is easy to do: we will just need the budget constraint and optimality conditions that link optimal consumptions in adjacent periods. Using our perturbation argument in class, we know that once we chose the optimal sequence $\{c_0^*, c_1^*, c_2^*, \ldots\}$, there will be no benefit, on the margin, to adjust consumptions in adjacent periods in a feasible manner. Thus, the optimal consumption sequence should satisfy a sequence of the following optimality (Euler) conditions:

$$
\begin{aligned}
MU_1 &= (1+r)MU_2 \\
MU_2 &= (1+r)MU_3 \\
MU_3 &= (1+r)MU_4 \\
MU_4 &= (1+r)MU_5 \\
&\vdots
\end{aligned}
$$

For our utility function, $U(c_0, c_1, c_2, \ldots)$, $MU_1 = \beta(\overline{c} - c_1)$, $MU_2 = \beta^2(\overline{c} - c_2)$, etc. Thus, at the optimum, the following set of equation should be satisfied:

$$
\begin{aligned}
\overline{c} - c_0^* &= \beta(1+r)(\overline{c} - c_1^*) \\
\beta(\overline{c} - c_1^*) &= \beta^2(1+r)(\overline{c} - c_2^*) \\
\beta^2(\overline{c} - c_2^*) &= \beta^3(1+r)(\overline{c} - c_3^*) \\
&\vdots \\
\sum_{t=0}^{\infty} \frac{c_t^*}{(1+r)^t} &= \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}.
\end{aligned}
$$

Since we assume that $\beta = \frac{1}{1+r}$, we can write this set of equations as:

$$
\begin{aligned}
c_0^* &= c_1^* \\
c_1^* &= c_2^* \\
c_2^* &= c_3^* \\
&\vdots \\
\sum_{t=0}^{\infty} \frac{c_t^*}{(1+r)^t} &= \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}.
\end{aligned}
$$

Thus, at the optimum, $c_0^* = c_1^* = c_2^* = c_3^* = \ldots = c^*$: for these preferences, consumer will choose to perfectly smooth consumption over time. Utilizing the budget constraint, $c^* \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}$.[3] Thus, $c^* = c_0^* = c_1^* = c_2^* = \ldots = \underbrace{\frac{r}{1+r}\left[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}\right]}_{y^p}$, where y^p is an estimate of the permanent income. Intuitively, it means that the consumer can sell his endowments forward (i.e., transfer the property rights on the endowment stream to, say, some financial institution), receiving $\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}$ in total, put this amount into a bank, and consume the interest on this amount each period forever. Sometimes y^p is called the annuity value of the present discounted sum of the income/endowment stream. This was one of the main insights of Milton Friedman, that individual consumption in each period should be related to an estimate of the permanent income.

If income is equal in each period so that $y_0 = y_1 = y_2 = \ldots = \overline{y}$, then the permanent

[3] Note that $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \ldots$ is an infinite geometric series with each successive term obtained as the previous term in the sequence multiplied by $\frac{1}{1+r} < 1$. Thus we want to find $S = 1 + a + a^2 + a^3 + \ldots$ if $|a| < 1$, where $S = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t}$, and $a = \frac{1}{1+r}$. The final result is $S = \frac{1}{1-a}$. Thus, $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1}{1-\frac{1}{1+r}} = \frac{1+r}{r}$.

income in each period is equal to $\frac{r}{1+r}\overline{y}\left[1+\frac{1}{1+r}+\frac{1}{(1+r)^2}+\frac{1}{(1+r)^3}+\dots\right]=\overline{y}\frac{r}{1+r}\frac{1+r}{r}=\overline{y}$. Thus, consumption in each period will be constant and equal to $\overline{y}$, the latter being the constant flow of a consumer's income and the permanent income.

In reality, if the current period is t, the consumer does not know with *certainty* his endowment/income stream after time t, $\{y_{t+1}, y_{t+2}, y_{t+3}, \dots\}$. In this case, it does not make sense to set consumptions for periods c_{t+1}, $c_{t+2}, \dots$ once and for all, since new information about future incomes and permanent income will arrive in periods following t (e.g., a consumer may receive an unexpected raise in period $t+1$ that will be permanent, or win a huge sum in the lottery in period $t+1$ that will raise his unearned income and permanent income). In this case, the optimality (Euler) condition that links optimal consumptions in periods t and $t+1$, for the utility function we adopted, will read as: $c_t^* = E_t c_{t+1}^*$,where $E_t c_{t+1}^*$ denotes expectation of consumption in period $t+1$ given all the available information in period t. Since we are taking expectation of c_{t+1}, we are assuming that consumption in period $t+1$ is not fully predictable. It is not fully predictable because we do not know with certainty the permanent income at $t+1$ (since we do not know with certainty our incomes $\{y_{t+1}, y_{t+2}, \dots\}$, given all the available information at time t). Note that Euler equation can be written as $E_t\left(c_{t+1}^* - c_t^*\right) = E_t \Delta c_{t+1}^* = 0$. It means that the expected future change in consumption, given all the available information at time t, is equal to zero, that is consumption does not change between periods t and $t+1$ if there is no additional information arriving between periods t and $t+1$ about consumer's incomes. In statistics, a variable that has this property is called a *martingale*. An implication of the martingale property of consumption is that consumption in period $t+1$ will differ from consumption in period t *only if* a consumer receives unexpected "news" about his permanent income. Thus, if a consumer knows at time t that income at time $t+1$ will be unusually low, he will adjust consumption now and will not be waiting until he sees the income drop (e.g., he may borrow now to smooth consumption in anticipation of the income drop). The result is due to the fact that, for the preferences we're using, a consumer will want to smooth consumption in adjacent periods.

In terms of the levels of consumption, we may derive the following relationship: $c_t = y_t^p = E_t\left[\frac{r}{1+r}\left(y_t + \frac{y_{t+1}}{1+r} + \frac{y_{t+2}}{(1+r)^2} + \frac{y_{t+3}}{(1+r)^3} + \ldots\right)\right]$. The logic is the same as before: consumption at time t is equal to an estimate of the permanent income, equal to the expectation of the annuity value of the presented discounted sum of incomes at times t, $t+1$, $t+2$, ...

The last important implication is that consumption will adjust by a larger margin if an unexpected change in income is permanent. Thus, if, unexpectedly, a consumer becomes permanently disable and less productive in period $t+1$, and the disability reduces his incomes in periods $t+1$, $t+2$, $t+3$, etc. by the same magnitude, then he will reduce consumption in period $t+1$ by the amount of the income drop. Intuitively, there is no way to "smooth" this permanent income drop, and so the consumption should be reduced by the amount of the drop. If income becomes unexpectedly low in one of the periods and the event, causing the income drop (say, a short spell of unemployment), does not affect much (or at all) incomes in other future periods, then the permanent income will not change much due to this unexpected "shock." Since the permanent income does not change much, consumption will not change much as well (a consumer will be able to borrow in order to smooth out this temporary shock).

All of these insights lead to the following conclusion, you've seen a lot in your previous classes. If the government contemplates about some policy affecting individual incomes (say, a tax cut) and wants to boost the economy via an increase in the aggregate consumption, it will only succeed if the policy affects permanent incomes a lot (say, a permanent reduction in income taxes). Otherwise, the reaction of consumers will be weak, if any.