The Phillips Curve, Rational Expectations, and the Lucas Critique

Instructor: Dmytro Hryshko

Outline

- Phillips curve as the short-run tradeoff between inflation and unemployment: inflation surprises lead to a reduction in unemployment.
- There is no tradeoff in the long run
- The importance of expectations (adaptive and rational)
- Rational expectations and the Lucas critique of policy evaluation

Phillips Curve

Models of Short-Run Aggregate Supply

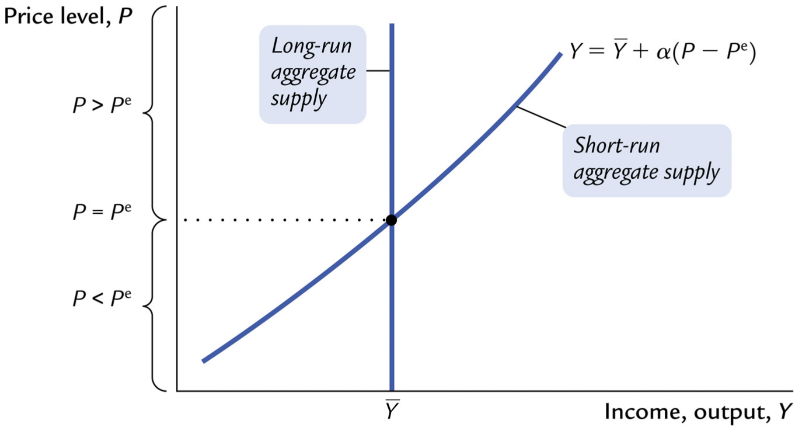
Common to all models (the sticky wage, the sticky price, and imperfect information models):

- Some friction/market imperfection causes output to deviate from the natural level.
- SRAS curve can be expressed as:

$$Y = \overline{Y} + \alpha \times (P - P^e),$$

where $\overline{Y}$ is the natural level of output, and

P^e is the expected level of prices, and $\alpha > 0$.


Price level, P
$P > P^e$
$P = P^e$
$P < P^e$
Long-run aggregate supply
$Y = \overline{Y} + \alpha(P - P^e)$
Short-run aggregate supply
$\overline{Y}$
Income, output, Y

Inflation, Unemployment, and the Phillips Curve

The Phillips curve reflects the tradeoff between unemployment and inflation: as policymakers move the economy along the SRAS, unemployment and inflation move in opposite directions.

$$P = P^e + (1/\alpha) \cdot (Y - \overline{Y}) + \nu \tag{SRAS}$$

$$P - P_{-1} = (P^e - P_{-1}) + (1/\alpha) \cdot (Y - \overline{Y}) + \nu$$

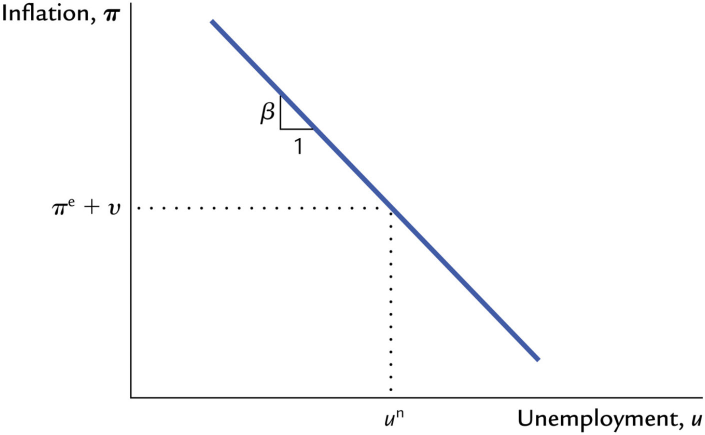
$$\pi = \pi^e + (1/\alpha) \cdot (Y - \overline{Y}) + \nu \tag{log-rule}$$

$$(1/\alpha) \cdot (Y - \overline{Y}) = -\beta \cdot (u - u^n) \tag{Okun's law}$$

$$\pi = \pi^e - \beta \cdot (u - u^n) + \nu,$$

where ν is the supply shock, and u^n is the natural rate of unemployment.

Short-run Phillips Curve: $\pi = \pi^e - \beta \cdot (u - u^n) + \nu$



Given π^e, β measures the tradeoff between inflation and unemployment in the short-run.

Adaptive Expectations and Inflation Inertia

Phillips Curve:

$$\pi = \pi^e - \beta \cdot (u - u^n) + \nu.$$

What determines π^e?

Adaptive expectations: $\pi^e = \pi_{-1}$.

Phillips curve under adaptive expectations

$$\pi = \pi_{-1} - \beta \cdot (u - u^n) + \nu.$$

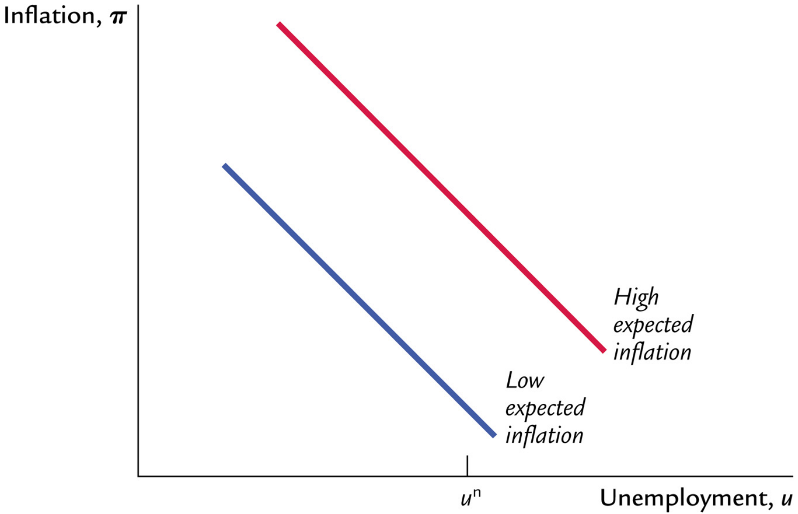
Inflation reduction is painful—entails increased unemployment and lost output.

The Short Run Tradeoff Between Inflation and Unemployment

The Phillips curve is drawn for a given π^e, and represents the short-run policymaker's menu of inflation/unemployment.

If π^e rises, the Phillips curve shifts upward and the menu is less attractive: for a given unemployment rate, inflation rate is higher.

In the long run, inflation adapts to the inflation rate chosen by the policymaker, and $u = u^n$ (PC is vertical in the long-run).


Inflation, π
High expected inflation
Low expected inflation
u^n
Unemployment, u

Phillips curve in the data

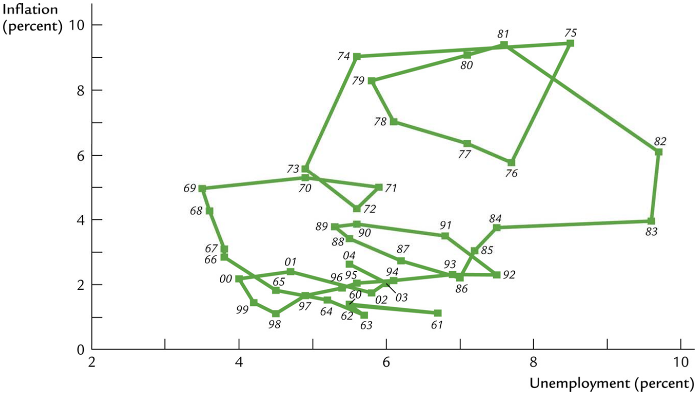


Figure 13.5 Inflation and Unemployment in the United States Since 1960
Mankiw: Macroeconomics, Sixth Edition

Rational Expectations and the Possibility of Painless Disinflation

If firms and households form rational expectations (RE), i.e., adjust their expectations to credible policies and announcements, inflation will exhibit less inertia.

- RE: short run tradeoff is not an accurate description of the policymaker's menu.
- RE: at the extreme, disinflation may be costless if done correctly, i.e., if policies are announced beforehand, and if they are credible.

Rational and Adaptive Expectations

Adaptive Expectations

- We formulate expectations today, on the basis of what happened yesterday about some outcome (interest rates, inflation, etc).
- Consider a stationary process x_t. Denote by $x^e_{t,t+1}$ an expectation for tomorrow's x formed on the basis of information available in period t (today).
- What do we know today?
  - what happened today, x_t;
  - what we thought would happen, $x^e_{t-1,t}$, on the basis of the information we had yesterday (at time $t-1$).

Adaptive expectations

Can we use these 2 pieces of information to formulate a new expectation? Take a weighted average to create:

$$x^e_{t,t+1} = \gamma x_t + (1-\gamma) x^e_{t-1,t}, \qquad 0 \leq \gamma \leq 1$$

or write it as the revision to the expectation

$$x^e_{t,t+1} - x^e_{t-1,t} = \gamma(\underbrace{x_t - x^e_{t-1,t}}_{\text{forecast error}})$$

From Adaptive to Rational Expectations

$$x^e_{t,t+1} - x^e_{t-1,t} = \gamma(\underbrace{x_t - x^e_{t-1,t}}_{\text{forecast error}})$$

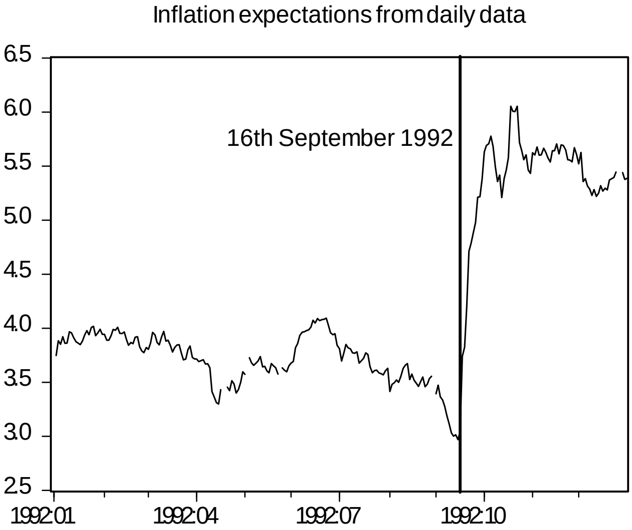
- Our expectation is revised proportionally to the difference between our previous expectation and the actual outcome. Expectations are adaptive (backward-looking, adapted to our past forecast errors).
- However, suppose we work in the financial markets and we are trying to forecast inflation. Would we only use information on past outcomes and expectations of inflation? Suppose monetary policy is invested in an independent central bank and the bank has a target for inflation. Should we take this into account? Should we be more forward looking?

From Adaptive to Rational Expectations

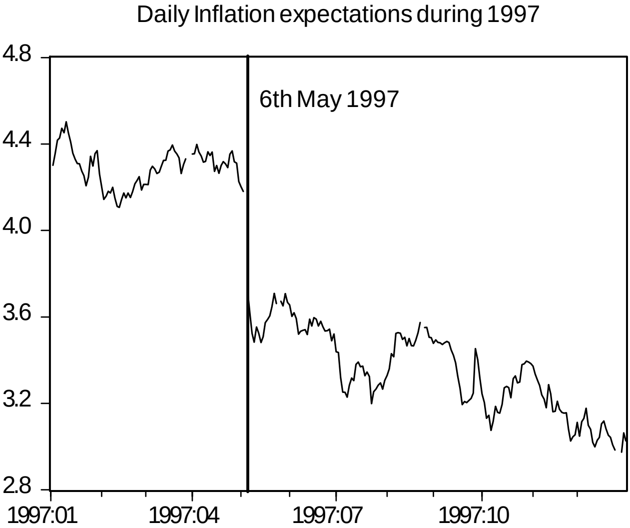
- Maximizing behavior is one of the corner stones of economic theory. It seems entirely natural that when forming expectations individuals should seek to use all of the information available in order to minimize the forecast error, $x_t - x^e_{t-1,t}$.

- Of course it depends on what we mean by all available information.

Two examples of forward looking behavior. Black Wednesday, England withdraws from the ERM



Operational responsibility for setting interest rates is granted to the BoE



Rational expectations

- It seems entirely natural to think that in formulating expectations we use all of the information that is available, including views of what we believe governments and central banks might do in the future.
- But how can we formalise this? Let us treat expectations as rational.
- Muth (1961) is usually credited with first suggesting the use of rational expectations.

Rational expectations

- Expectations "are essentially the same as the predictions of the relevant economic theory." (Negishi, 1964)
- The expectations of economic agents should be consistent with the models used to explain their behavior.

More on RE: Nerlove

- RE does not require that every farmer or business-man formulate a correct and relevant economic model
- RE requires the representative firm behaves *as if* it had made predictions on the basis of the same economic model used by the economist to analyze industry behavior
- Expectations are constructs of the same nature as "certainty equivalents," "supply functions," etc.
- If expectations were not rational, at least on the average, then insofar as our economic model approximates reality we should tend to find a small group of individuals, whose expectations are better than those of the rest, gradually driving the others out of business.

More on RE: Prescott

"...like utility, expectations are not observed, and surveys cannot be used to test the Rational Expectations hypothesis. One can only test if some theory, whether it incorporates Rational Expectations or for that matter, irrational expectations, is or is not consistent with observations."

Rational Expectations

- For a discrete random variable x with outcomes $i, i+1, \ldots, n$ and probability of event i equal to P_i, we have the expected value of x

$$E(x) = \sum_{i}^{n} P_i x_i.$$

- For a continuous random variable X we have

$$E(x) = \int_a^b x f(x) dx.$$

Rational expectations

We want the conditional expectation. We make an assessment of an outcome on the basis of the information available to us at any moment in time. Denote this information set available at time $t-1$ as I_{t-1} (a vector of relevant variables). The conditional expectation of x based on I_{t-1} can be written

$$E\left[x_t \mid I_{t-1}\right] = \int_a^b x_t f\left(x_t \mid I_{t-1}\right) dx_t,$$

where $f(x_t \mid I_{t-1})$ is the conditional probability density for the random variable, x_t.

Rational expectations

Associated with the expectation is a forecast error, $\epsilon_t = x_t - E[x_t \mid I_{t-1}]$, with 2 central properties.

1. The expectation of the error is zero. There is no bias.

$$E[\epsilon_t \mid I_{t-1}] = 0 \quad (= E[x_t \mid I_{t-1}] - E[x_t \mid I_{t-1}])$$

2. The error is independent of the underlying information set. Otherwise there is information that is not been utilized in order to produce the 'best' forecast.

$$E[\epsilon_t \cdot I_{t-1} \mid I_{t-1}] = 0$$

Muth's hypothesis : $\underbrace{x^e_{t-1,t}}_{\text{subjective expectation}} = \underbrace{E[x_t \mid I_{t-1}]}_{\text{conditional mathematical expectation}}$

Question

What are the implications of this for macroeconomics and the conduct of macroeconomic policy?

Lucas Critique

Lucas critique (1976)

> *...the "long-run" implications of current forecasting models are without content, and [that] the short-term forecasting ability of these models provides no evidence of the accuracy to be expected from simulations of hypothetical policy rules."*

Econometric models of policy evaluation: assume that the structure of the economy doesn't change when different policy scenarios are imposed.

Lucas (1976): this can be very misleading!

Lucase critique. Example: Hall's consumption function

- Assume quadratic utility $u(c_t) = ac_t - \frac{b}{2}c_t^2$; the time discount factor, β, equals $\frac{1}{1+r}$, where r is the real interest rate.
- Present value budget constraint:

$$\sum_{j=0}^{\infty} E_t \beta^j c_{t+j} = A_t + \sum_{j=0}^{\infty} E_t \beta^j y_{t+j},$$

where, e.g., $E_t y_{t+1}$ denotes the conditional expectation of y_{t+1} (based on information at t), c_t consumption, y_t income, and A_t wealth at t.

PIH and the Lucas critique

- Euler equation: $E_t c_{t+k} = c_t$, $k \geq 1$.

$$c_t = (1-\beta)\left[A_t + \underbrace{\sum_{j=0}^{\infty} E_t \beta^j y_{t+j}}_{?}\right]$$

- Assume $A_t = 0$. Need to make predictions about $y_{t+1}, y_{t+2}, \ldots$ based on information available at t (today).
- Easy if the process "generating" y_t is known (take mathematical expectation).

Lucas critique. Example: Hall's consumption function

- Assume income is a random walk: $y_{t+1} = y_t + \epsilon_{t+1}$, where ϵ_{t+1} is white noise. Then

$$E_t[y_{t+1}] = E_t[\underbrace{y_t + \epsilon_{t+1}}_{=y_{t+1}}] = y_t + E_t[\epsilon_{t+1}] = y_t$$

$$E_t[y_{t+2}] = E_t[\underbrace{y_{t+1}}_{=y_t+\epsilon_{t+1}} + \epsilon_{t+2}] = y_t + E_t[\epsilon_{t+1}] + E_t[\epsilon_{t+2}] = y_t$$

$$\vdots$$

$$E_t[y_{t+k}] = y_t, \quad \text{all } \ k \geq 1.$$

Therefore

$$\sum_{j=0}^{\infty} E_t \beta^j y_{t+j} = y_t + \beta y_t + \beta^2 y_t + \ldots = \frac{y_t}{1-\beta}.$$

Lucas critique

It follows that the consumption rule when income is random walk (permanent shocks) is $c_t = y_t$ (consume what you get).

- Econometrician will estimate $c_t = \kappa_0 + \kappa_1 y_t$, where $k_0 = 0$ and $\kappa_1 = 1$.
- Now assume a public policy eliminates persistence of the shocks to income: $y_{t+1} = \bar{y} + \epsilon_{t+1}$. Following the same steps as before, the consumption rule will be modified to

$$c_t = \bar{y}.$$

- The policy changes the environment, expectations, and consumers' behavior. Using the previously estimated rule $c_t = \kappa_0 + \kappa_1 y_t$ will give wrong predictions on c_t under the new policy regime.
- Forecasts relying on historical data can be quite misleading!

Suggested readings

- Sheffrin, S. *Rational Expectations*, Cambridge University Press, Ch. 1–3.
- Warren Young and William Darity Jr. 2001. *The Early History of Rational and Implicit Expectations*. History of Political Economy. `http://muse.jhu.edu/journals/history_of_political_economy/v033/33.4young.pdf`
- Mankiw and Scarth. Fifth Canadian Edition. Chapter 13 (Phillips curve), Chapter 15 (Lucas Critique).