Search and Unemployment

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Outline

- Data: unemployment rate, and participation rate
- Key determinants of the unemployment rate: aggregate economic activity (cycles), demographics (young more likely to be unemployed), government intervention (UI system), mismatch (sectoral shifts)
- The Diamond-Mortensen-Pissarides (DMP) Model of Search and Unemployment
Labor market data

- $N =$ working-age population (Labor force + Out of labor force)
- $Q =$ labor force ($L$ in terms of the notation we used in the last lecture=employed+unemployed)
- $U =$ number of unemployed
- Unemployment rate $= U/Q$
- Participation rate $= Q/N$
Cyclical behavior of unemployment rate and GDP in Canada

Unemployment rate is countercyclical (above the trend when GDP is below its trend).
Participation rate in Canada
Participation rate of men and women in Canada
Participation rate and GDP

Participation rate is procyclical (above the trend when GDP is above its trend).
The DMP Search Model of Unemployment

- Optimizing Consumers
- Optimizing Firms
- Matching in the labor market
- Equilibrium
Consumers

- Each consumer chooses between home production (be part of N but not Q) and searching for work (be part of Q)
- Different consumers place different values on working outside the market
- Supply curve of consumers searching for work, $v(Q)$
- $v(Q)$ denotes the expected payoff from searching for work that would induce $Q$ consumers to search versus remaining outside the labor force
The supply curve is upward sloping because different consumers have different payoffs to working at home; more consumers join the labor force if the payoff gets higher.
Firms

- There are many firms that could potentially be active in searching in the labor market.
- It costs a firm $k$ to post a vacancy and advertise an open position.
- $A =$ the number of active firms posting vacancies.
Matching

- The matching process in the labor market is described by the matching function

\[ M = em(Q, A), \]

where

- \( M \) = number of successful matches
- \( Q \) = number of consumers searching for work
- \( A \) = number of firms searching for workers
- \( e \) = matching efficiency
Matching function: \( M = em(Q, A) \)

- The matching function works like a production function
- Matches \( M \) are the “output” (output of the matching process)
- \( Q \) and \( A \) are the “factor inputs”
- \( e \) plays the role of “total factor productivity” (will enable us to look at cyclical behavior of unemployment)
- \( m \) is CRS in \( Q \) and \( A \):

\[
xM = em(xQ, xA)
\]

- \( m(0, A) = m(Q, 0) = 0 \) — no matches with no people searching for work, or no firms searching for workers
- \( M \) is increasing in \( Q \) and \( A \) (positive marginal products)
- Marginal products are diminishing
Probability of Finding Work for a Consumer

- Let $p_c$ be the probability of finding work for a consumer = Number of successful matches divided by the number of consumers searching for work = $M/Q$
- We assumed $m$ to be CRS: $xM = em(xQ, xA)$. Let $x = \frac{1}{Q}$ to obtain

$$p_c = \frac{M}{Q} = em \left(1, \frac{A}{Q}\right) = em(1, j), \quad p_c \in (0, 1)$$

where $\frac{A}{Q} = j$ is the labor market tightness, the number of actively searching firms per searching worker.

Note that $m$ (and therefore $p_c$) is increasing in $j$ by our assumption of positive marginal products.

- The probability of being unemployed if a consumer chooses to search equals

$$1 - p_c = 1 - em(1, j), \quad (1 - p_c) \in (0, 1)$$

and is decreasing in $j$. 
Consumer Optimization

- $Q$ consumers search for work, and $v(Q)$ is supply curve for the number of consumers choosing to search for work.
- If consumers work, they receive wage from working $w$, if unemployed (actively engaged in search but have no work), they receive an employment insurance benefit $b$.
- In equilibrium, the expected payoff from searching must equal $v(Q)$.

$$v(Q) = p_c w + (1 - p_c)b = b + p_c(w - b) = b + em(1, j)(w - b)$$
Determination of the Labor Force, $Q$

The market wage, the EI benefit, and labor market tightness determine the expected payoff to searching. Then, given this payoff, the supply curve for searching consumers determines the labor force $Q$. 

![Graph showing the relationship between expected payoff to searching for work and the labor force $Q$.]
Probability of a Successful Match for an Active Firm

- Firms bear cost \( k \) of posting a vacancy
- Face the probability \( p_f = \frac{M}{A} \) of finding a worker
- Use the CRS property of the matching function \( xM = em(xQ, xA) \) and let \( x = \frac{1}{A} \) to obtain

\[
p_f = \frac{M}{A} = em \left( \frac{Q}{A}, 1 \right) = em \left( \frac{1}{j}, 1 \right), \quad p_f \in (0, 1).
\]

The probability of filling a vacancy for a firm is decreasing in labor market tightness \( j \).
Optimization by firms

- The firm-worker pair produce output $z$. Let the price of output be normalized to 1, and assume no capital is used for production.
- The firm’s profit is $z - w$.
- Firms will enter the market and post vacancies until the expected payoff from doing this is zero:

$$p_f(z - w) - k = 0$$

$$\Rightarrow em\left(\frac{1}{j}, 1\right) (z - w) = k$$

$$\Rightarrow em\left(\frac{1}{j}, 1\right) = \frac{k}{z - w}$$
Determination of the labor market tightness, $j$

Firms post vacancies until the probability for a firm of matching with a worker equals the ratio of the cost of posting a vacancy to the profit the firm receives from a successful match.

$j = \text{Labour Market Tightness}$
Match surplus

- In a successful match, the worker and the firm will produce $z$.
- Successful match generates a surplus:

  \[
  \text{Firm’s surplus} = z - w \\
  \text{Worker’s surplus} = w - b
  \]

  \[
  \text{Total surplus} = (z - w) + (w - b) = z - b
  \]

- Need to determine the wage, but this is not a competitive market! In a competitive market, $w = z$. 
Nash Bargaining solution

Assume that the worker gets a fraction \( a \) of the total surplus from a successful match:

\[
w - b = a(z - b),
\]

while the firm gets the fraction \( 1 - a \) of the total surplus:

\[
z - w = (1 - a)(z - b).
\]

This is the solution of the Nash bargaining problem:

\[
\max(w - b)^a(z - w)^{1-a}
\]

s.t.

\[
S = w - b + z - w = z - b.
\]

The wage equation can be rewritten as:

\[
w = az + (1 - a)b.
\]
Equilibrium

Two equations determining $Q$ and $j$:

\[ v(Q) = b + em(1, j)(w - b) = b + em(1, j)a(z - b) \]

\[ em \left( \frac{1}{j} , 1 \right) = \frac{k}{z - w} = \frac{k}{(1 - a)(z - b)} \]

- The second equation determines $j$ given $a, k, z, b$ (exogenous variables);
- then, given $j$, the first equation determines the labor force $Q$. 
Equilibrium in the DMP model

In panel (b), the ratio of the cost of a vacancy to the firm’s surplus from a successful match determines labor market tightness, $j$. Then, in panel (a), labor market tightness determines the size of the labor force $Q$. 

$$\nu(Q) = b + e m(1, j)a(z - b)$$
Equilibrium quantities

Unemployment rate:

\[ u = \frac{U}{Q} = \frac{(1 - p_c)Q}{Q} = 1 - p_c = 1 - em(1, j). \]

Vacancy rate (=number of unfilled jobs relative to the number of firms \( A \) (potential jobs)):

\[ v = \frac{(1 - p_f)A}{A} = 1 - p_f = 1 - em \left( \frac{1}{j}, 1 \right). \]

Aggregate output:

\[ Y = Mz = zem(Q, A) \]
\[ xY = zem(xQ, xA) \]
\[ \Rightarrow \frac{Y}{Q} = zem \left( 1, \frac{A}{Q} \right) = zem(1, j) \]
\[ \Rightarrow Y = Q \cdot z \cdot e \cdot m(1, j). \]
Next: various experiments

1. An increase in the unemployment insurance benefit, $b$ (propping up incomes in recessions)
2. An increase in productivity, $z$ (expansion)
3. A decrease in matching efficiency, $e$ (sectoral mismatch)
Equations to keep track of

Eqm equations:

\[
\begin{align*}
v(Q) &= b + em(1, j)(w - b) = b + em(1, j)a(z - b) \\
em \left( \frac{1}{j}, 1 \right) &= \frac{k}{z - w} = \frac{k}{(1-a)(z-b)}
\end{align*}
\]

Outcomes of interest:

\[
\begin{align*}
u &= 1 - p_c = 1 - em(1, j) \\
v &= 1 - p_f = 1 - em \left( \frac{1}{j}, 1 \right) \\
Y &= Q \cdot z \cdot e \cdot m(1, j)
\end{align*}
\]
An Increase in the Employment Insurance Benefit, $b$

- Lowers the total surplus, $z - b$, and the firm’s surplus
  $$(1 - a)(z - b) \Rightarrow \text{less attractive to post vacancies} \Rightarrow j \downarrow$$
- Two counteracting effects on $Q$: higher $b$ encourages consumers to search, but the chances of finding a job are lower (since lower $j$) $\Rightarrow$ the end effect on labor force $Q$ is ambiguous. In the diagram below, the curves are drawn in such a way that $Q$ falls
- Unemployment rate unambiguously rises
- Vacancy rate unambiguously falls
- The effect on output ambiguous; output falls for sure if $Q$ falls
An increase in the EI benefit, $b$

An increase in $b$ reduces the surplus the firm receives from a match, which reduces labor market tightness in panel (b). Then, in panel (a), the increase in $b$ shifts the curve up. The labor force could decrease or increase.
An Increase in Productivity, $z$

- Increases the total surplus, $z - b$, and the firm’s surplus $(1 - a)(z - b) \Rightarrow$ more attractive to post vacancies $\Rightarrow j \uparrow$
- Higher $z$ encourages more consumers to enter the labor market, since the chances of finding a job are higher ($j \uparrow$) and wages are higher (more surplus to be shared between the firm and worker) $\Rightarrow$ labor force $Q \uparrow$
- Unemployment rate unambiguously falls
- Vacancy rate unambiguously rises
- Output unambiguously increases
- These predictions are consistent with the comovements in labor market variables over the business cycle (unemployment rate is countercyclical while the participation rate is procyclical)
An increase in productivity, $z$

An increase in $z$ increases the surplus from a match for both workers and firms. In panel (b), labor market tightness $j$ increases, and the curve shifts up in panel (a), such that the labor force $Q$ must increase.
A Decrease in Matching Efficiency, $e$

- Chances of a successful match fall for a firm, so labor market tightness $j$ must fall.
- Labor force $Q$ falls because the lower matching efficiency lowers the probability of a match and results into a fewer firms searching (lower $j$).
- Unemployment rate unambiguously increases since both $e$ and $j$ fall.
- Vacancy rate stays constant. Two counteracting effects: $e \downarrow$ vacancy rate $\uparrow$, $j \downarrow \Rightarrow$ vacancy rate $\uparrow$, but the second equilibrium equation tells us that $em\left(\frac{1}{j}, 1\right)$ must stay constant and so the two effects balance each other out.
- Output unambiguously falls since $e$, $j$, and $Q$ all fall.
A decrease in matching efficiency, $e$

This acts to shift the curves down in panels (b) and (a). Labor market tightness and labor force must both decrease.
The Beveridge Curve for the U.S.

What causes the observed Beveridge relationship?

1. Variation in matching efficiency, $e$? No, since the vacancy rate stays constant with changes in $e$.

2. Variation in EI benefit, $b$? Potentially yes, but unlikely in practice since there’re no large and frequent changes in $b$ in the data.

3. Variation in productivity, $z$? Yes, a good candidate.

Notice that an increase in productivity (leads to low unemployment rate and high vacancy rate) and a decrease in matching efficiency (leads to a stable vacancy rate but high unemployment rate) can explain a shift in the Beveridge curve in the aftermath of the Great Recession (the observed high vacancy rate but stable unemployment rate).
Performance of the DMP model during the Great Recession 2007–2009 and after it
DMP prediction: unemployment rate should recover faster, and output should recover faster
DMP prediction is violated
DMP prediction is violated
What’s wrong?

The degree of mismatch in the U.S. labor market (reduction in $e$) might have increased much more in the U.S. than in Canada from the beginning of 2008 to mid-2011 (can be traced in the U.S. to the dramatic drop in construction and sectoral shifts in the U.S., while Canada experienced only a moderate decline in construction).

Thus, the DMP model is fine if the extent of labor market mismatch is taken into account.
Readings