The Permanent Income Hypothesis (PIH)

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A 2-period formulation

- 2-period problem, periods 0 and 1.
- Within-period (instantaneous) utility function is quadratic:

$$u(c_t) = -\frac{1}{2}(c_t - \bar{c})^2.$$

- Freely borrow/lend at the constant real interest rate r.
- Endowments y_0 and y_1 are known at time 0.
- $\bar{c}$ is the "bliss" consumption level. If $c_t = \bar{c}$, a consumer attains the maximum utility possible, equal to 0.
- $\bar{c} \geq c_t$ so that the marginal utility is positive: $\bar{c} - c_t > 0$.
- $\beta \in (0, 1)$ is the time discount factor.
- $\beta(1 + r) = 1$.

Optimization problem

$$\max_{c_0 \geq 0, c_1 \geq 0} U(c_0, c_1) = -\frac{1}{2}(c_0 - \overline{c})^2 - \beta \frac{1}{2}(c_1 - \overline{c})^2$$
$$\text{s.t. } c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r},$$

where $\beta \in (0, 1)$ is the time discount factor. For this utility function,

$$MU_0 = \bar{c} - c_0$$
$$MU_1 = \beta(\overline{c} - c_1).$$

Optimum

At the optimum, the following two equations should be satisfied:

$$(\bar{c} - c_0^*) = \beta(1 + r)(\bar{c} - c_1^*)$$

$$c_0^* + \frac{c_1^*}{1 + r} = y_0 + \frac{y_1}{1 + r}.$$

Since we assumed that $\beta = \frac{1}{1+r}$, we can write the first of those equations as

$$\bar{c} - c_0^* = \bar{c} - c_1^*, \quad \text{or} \quad c_0^* = c_1^*.$$

Plugging this equilibrium condition into the second equation, we obtain $c_0^* + \frac{c_0^*}{1+r} = y_0 + \frac{y_1}{1+r}$, or

$$c_0^* = c_1^* = \frac{1 + r}{2 + r}\left(y_0 + \frac{y_1}{1 + r}\right).$$

Consumer, *for these preferences*, will prefer to smooth consumption across periods perfectly.

Infinite horizon–1

Assume instead that a consumer's horizon is infinite, and s/he chooses consumption for periods $t = 0, 1, 2, \ldots$ In this case,

$$\max_{c_0 \geq 0, c_1 \geq 0, c_2 \geq 0, \ldots} U(c_0, c_1, c_2, \ldots) = \underbrace{-\frac{1}{2}(c_0 - \overline{c})^2}_{=u(c_0)} + \beta \left[\underbrace{-\frac{1}{2}(c_1 - \overline{c})^2}_{=u(c_1)} \right]$$

$$+ \beta^2 \left[\underbrace{-\frac{1}{2}(c_2 - \overline{c})^2}_{=u(c_2)} \right] + \beta^3 \left[\underbrace{-\frac{1}{2}(c_3 - \overline{c})^2}_{=u(c_3)} \right] + \ldots$$

$$\text{s.t. } c_0 + \frac{c_1}{1+r} + \frac{c_2}{(1+r)^2} + \frac{c_3}{(1+r)^3} + \ldots = y_0 + \frac{y_1}{1+r} + \frac{y_2}{(1+r)^2} + $$

$$+ \frac{y_3}{(1+r)^3} + \ldots$$

Infinite horizon–2

More compactly,

$$\max_{c_0 \geq 0, c_1 \geq 0, c_2 \geq 0, \ldots} U(c_0, c_1, c_2, \ldots) = \sum_{t=0}^{\infty} \left[-\frac{1}{2} \beta^t (c_t - \overline{c})^2 \right]$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}.$$

Now, instead of finding just c_0^* and c_1^*, we will need to find the whole (infinite) sequence $\{c_0^*, c_1^*, c_2^*, \ldots\}$.

Not so hard! Just need the (optimality) Euler equations and the lifetime budget constraint.

The equations to be satisfied at the optimum

$$
\begin{aligned}
MU_1 &= (1+r)MU_2 \\
MU_2 &= (1+r)MU_3 \\
MU_3 &= (1+r)MU_4 \\
MU_4 &= (1+r)MU_5 \\
&\vdots \\
\sum_{t=0}^{\infty} \frac{c_t^*}{(1+r)^t} &= \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}.
\end{aligned}
$$

In terms of our utility function, the following equations should be satisfied at the optimum:

$$\underbrace{\bar{c} - c_0^*}_{\text{MU}_0} = (1+r)\underbrace{\beta(\bar{c} - c_1^*)}_{\text{MU}_1}$$

$$\underbrace{\beta(\bar{c} - c_1^*)}_{\text{MU}_1} = (1+r)\underbrace{\beta^2(\bar{c} - c_2^*)}_{\text{MU}_2}$$

$$\underbrace{\beta^2(\bar{c} - c_2^*)}_{\text{MU}_2} = (1+r)\underbrace{\beta^3(\bar{c} - c_3^*)}_{\text{MU}_3}$$

$$\vdots$$

$$\sum_{t=0}^{\infty} \frac{c_t^*}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}.$$

Since we assume that $\beta = \frac{1}{1+r}$, the sequence of Euler equations implies

$$c_0^* = c_1^*, c_1^* = c_2^*, c_2^* = c_3^* \ldots \Rightarrow c_0^* = c_1^* = c_2^* = c_3^* = \ldots = c^*.$$

Plugging the result into the lifetime budget constraint,

$$c^* \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}.$$

Note that $\displaystyle\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \ldots,$ and $\frac{1}{1+r} < 1$. We want to find $S = 1 + x + x^2 + x^3 + \ldots$, where $x \equiv \frac{1}{1+r}$. This sum will be equal to $\frac{1}{1-x} = \frac{1}{1-\frac{1}{1+r}} = \frac{1+r}{r}$.

$$c^* = c_0^* = c_1^* = c_2^* = \ldots = \underbrace{\frac{r}{1+r}\left[\sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t}\right]}_{y^p}.$$

Milton Friedman: individual consumption in each period should be related to an estimate of the permanent income.

Aside

It is easy to show that

$$S = 1 + x + x^2 + x^3 + x^4 + \ldots = \frac{1}{1-x}, \text{for } \; |x| < 1.$$

Multiply the LHS and RHS of the equation by x,

$$xS = x + x^2 + x^3 + x^4 + x^5 + \ldots,$$

and subtract the result from S, to obtain

$$S - xS = (1 + x + x^2 + x^3 + x^4 + \ldots) - (x + x^2 + x^3 + x^4 + \ldots) = 1.$$

Thus,

$$S = \frac{1}{1-x}.$$

Example: constant flow of endowments

If $y_0 = y_1 = y_2 = \ldots = \overline{y}$, c^* will be equal to

$$\frac{r}{1+r}\overline{y}\left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \ldots\right] = \overline{y}\frac{r}{1+r}\frac{1+r}{r} = \overline{y}$$

Stochastic incomes

- In reality, future incomes are uncertain (that is, stochastic). At time t, when making consumption decision for time t, we do not know for sure $\{y_{t+1}, y_{t+2}, y_{t+3}, y_{t+4}, \ldots\}$.
- In this case, it does not make sense to set consumptions for periods c_{t+1}, $c_{t+2}, \ldots$ once and for all, since new information about future incomes and permanent income will arrive in periods following t.
- The optimality (Euler) condition that links optimal consumptions in periods t and $t+1$, for the utility function we adopted, will read as:

$$c_t^* = E_t(c_{t+1}^*),$$

where $E_t(\cdot)$ is expectation conditional on information (about future endowments) available at time t.

Stochastic Euler equation

Subtracting c_t^* from both sides,

$$E_t\left(c_{t+1}^* - c_t^*\right) = E_t \Delta c_{t+1}^* = 0.$$

It means that the expected future change in consumption, given all the available information at time t, is equal to zero, that is consumption does not change between periods t and $t+1$ if there is no additional information arriving between periods t and $t+1$ about consumer's incomes. In statistics, a variable that has this property is called a *martingale*.

An implication of the martingale property of consumption is that consumption in period $t+1$ will differ from consumption in period t *only if* a consumer receives unexpected "news" about his permanent income.

Optimal consumption with stochastic incomes

In terms of the levels of consumption, we may derive the following relationship:

$$c_t = y_t^p = E_t\left[\frac{r}{1+r}\left(y_t + \frac{y_{t+1}}{1+r} + \frac{y_{t+2}}{(1+r)^2} + \frac{y_{t+3}}{(1+r)^3} + \dots\right)\right].$$

Important implications

- Consumption will change between adjacent periods if (a consumer's estimate of) the permanent income changes
- Consumption will adjust by a larger margin if an unexpected change in income is permanent (e.g., compare disability vs. short spell of unemployment).
- If the government contemplates about some policy affecting individual incomes (say, a tax cut) and wants to boost the economy via an increase in the aggregate consumption, it will only succeed if the policy affects permanent incomes a lot (say, a permanent reduction in income taxes). Otherwise, the reaction of consumers will be weak, if any.