ECON 385. Intermediate Macroeconomic Theory II. Solow Model With Technological Progress and Data

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Examples of technological progress

- 1970: 50,000 computers in the world; 2000: 51% of U.S. households have 1 or more computers
- The real price of computer power has fallen an average of 30% per year over the past three decades
- The average car built in 1996 contained more computer processing power than the first lunar landing craft in 1969
- 1981: 213 computers connected to the Internet; 2000: 60 million computers connected to the Internet

Technological progress in the Solow model

- A new variable: E = labour efficiency
- Assume technological progress is labour-augmenting—it increases labour efficiency at the exogenous rate g:

$$\frac{\Delta E}{E} = g$$

- We now write the production function as

$$Y = F(K, EL)$$

where $L \times E =$ the number of effective workers (efficient units of labour).
- Hence, increases in labour efficiency have the same effect on output as increases in the labour force.

Notation

- $y = \frac{Y}{EL}$ = output per effective worker
- $k = \frac{K}{EL}$ = output per effective worker
- Production function per effective worker:
  $y = f(k)$
- Saving and investment per effective worker:
  $sy = sf(k)$

The law of motion of capital per effective worker

Start with $k \equiv \frac{K}{EL}$. Then,

$$\begin{aligned}
\frac{\Delta k}{k} &= \frac{\Delta K}{K} - \frac{\Delta E}{E} - \frac{\Delta L}{L} \\
&= \frac{I - \delta K}{K} - g - n \\
&= s\frac{Y}{K} - \delta - g - n \\
&= s\frac{Y/(EL)}{K/(EL)} - \delta - g - n \\
&= s\frac{y}{k} - \delta - g - n.
\end{aligned}$$

Multiplying both sides by k, we obtain

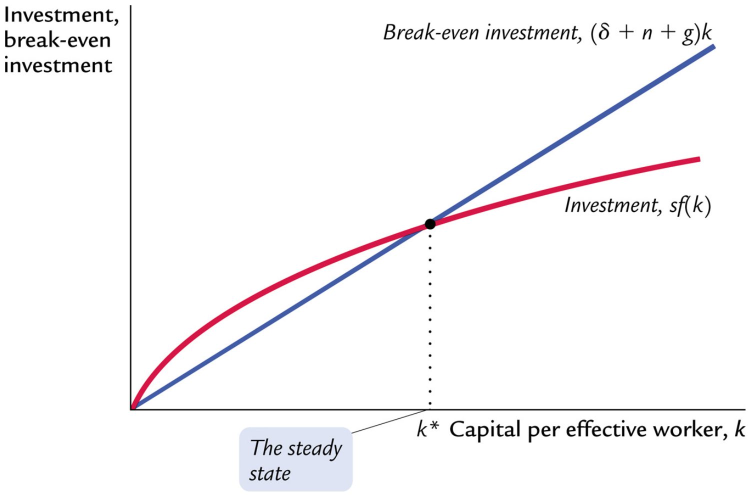
$$\Delta k = sy - (\delta + g + n)k = sf(k) - (\delta + g + n)k$$

Technological progress in the Solow model

$$\Delta k = sy - (\delta + g + n)k = sf(k) - \underbrace{(\delta + g + n)k}_{\text{break-even investment}}$$

Break-even investment consists of:

- δk to replace depreciating capital
- nk to provide capital for new workers
- gk to provide capital for the new "effective" workers "created" by technological progress



Investment, break-even investment
Break-even investment, $(\delta + n + g)k$
Investment, $sf(k)$
k^* Capital per effective worker, k
The steady state

Steady-State (balanced-path) Growth Rates in the Augmented Solow Model

Variable	Symbol	SS growth rate
Capital per effective worker	$k = \frac{K}{EL}$	0
Output per effective worker	$y = \frac{Y}{EL}$	0
Output per worker	$\frac{Y}{L} = yE$	g
Total output	$Y = yEL = \frac{Y}{L}L$	$n + g$

The Golden Rule

You need to maximize c^*

$$c^* = y^* - i^* = f(k^*) - (\delta + n + g)k^*$$

c^* is maximized when

$$\text{MPK} = \delta + n + g$$

Policies to promote growth

- Are we saving enough? Too much?
- What policies might change the saving rate?
- How should we allocate our investment between privately owned physical capital, public infrastructure, and "human capital"?
- What policies might encourage faster technological progress?

Evaluating the Rate of Saving

- Use the Golden Rule to determine whether our saving rate and capital stock are too high, too low, or about right.
- To do this, we need to compare (MPK $-\delta$) to $(n + g)$.
- If (MPK $-\delta$) $>$ $(n + g)$, then we are below the Golden Rule steady state and should increase s.
- If (MPK $-\delta$) $<$ $(n + g)$, then we are above the Golden Rule steady state and should reduce s.

Policies to increase the saving rate

Increase incentives for private saving:

- reduce capital gains tax, corporate income tax, estate tax as they discourage saving
- replace income tax with a consumption tax
- improve incentives for retirement savings accounts

Allocating the economy's investment

- In the Solow model, there's one type of capital
- In the real world, there are many types, which we can divide into three categories:
  –private capital stock
  –public infrastructure
  –human capital: the knowledge and skills that workers acquire through education
- How should we allocate investment among these types?

Allocating the economy's investment

- Equalize tax treatment of all types of capital in all industries, then let the market allocate investment to the type with the highest marginal product.
- Industrial policy: Government should actively encourage investment in capital of certain types or in certain industries, because they may have positive externalities (by-products) that private investors don't consider.

Encouraging technological progress

- Patent laws: encourage innovation by granting temporary monopolies to inventors of new products
- Tax incentives for R&D
- Grants to fund basic research at universities
- Industrial policy: encourage specific industries that are key for rapid technological progress

Growth empirics: Solow model against the facts

- Solow model's steady state exhibits balanced growth—many variables grow at the same rate
- Solow model predicts Y/L and K/L grow at same rate (g), so that K/Y should be constant. True in the real world.
- Solow model predicts real wage grows at same rate as Y/L, while real rental price is constant. True in the real world.

Table

TABLE 3-1

Growth in Labor Productivity and Real Wages: The U.S. Experience

Time Period	Growth Rate of Labor Productivity	Growth Rate of Real Wages
1959–2003	2.1%	2.0%
1959–1973	2.9	2.8
1973–1995	1.4	1.2
1995–2003	3.0	3.0

Source: Economic Report of the President 2005, Table B-49. Growth in labor productivity is measured here as the annualized rate of change in output per hour in the nonfarm business sector. Growth in real wages is measured as the annualized change in compensation per hour in the nonfarm business sector divided by the implicit price deflator for that sector.

Predictions and Empirics

- If the world behaves like the Solow model, we should observe convergence (in incomes per capita) if countries differ only with respect to initial capital and share same s, n, δ
- … then poor countries should grow faster (since they're farther away from SS) and we would expect a negative relationship between initial income and growth
- Do not observe such absolute convergence in a broad cross-section of countries as they differ in s, n and δ

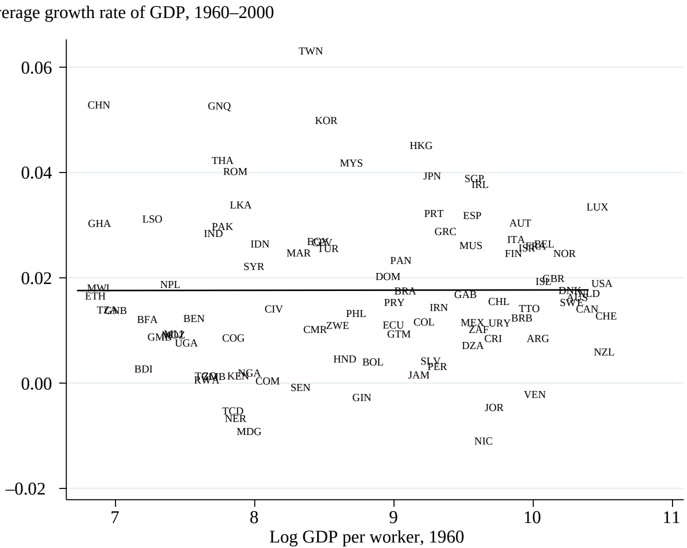


FIGURE 1.13 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for the entire world.

Source: Acemoglu (2008). Introduction to Modern Economic Growth

Convergence

- Many poor countries do NOT grow faster than rich ones. Does this mean the Solow model fails?
- No, because "other things" aren't equal.
- In samples of countries with *similar* savings & population growth rates, income gaps shrink about 2%/year

Conditional Convergence

- What the Solow model really predicts is conditional convergence—countries converge to their own steady states, which are determined by saving, population growth, and education
- And this prediction comes true in the data

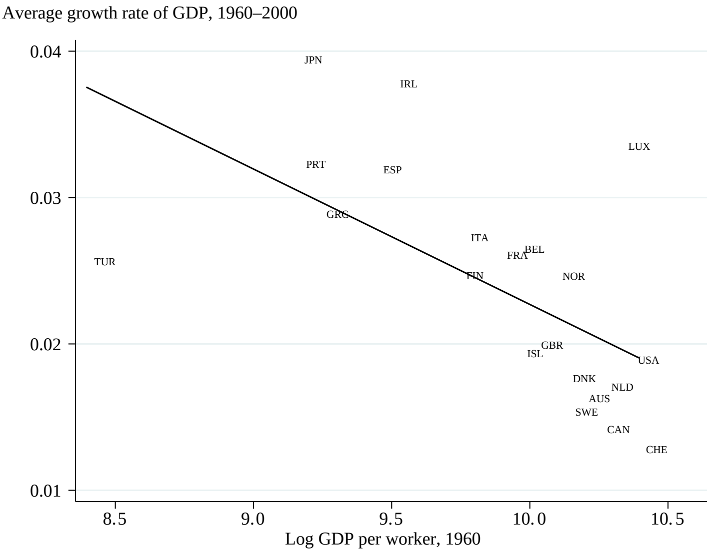


FIGURE 1.14 Annual growth rate of GDP per worker between 1960 and 2000 versus log GDP per worker in 1960 for core OECD countries.

Source: Acemoglu (2008). Introduction to Modern Economic Growth

More on convergence

$$\Delta k = s\,f(k) - (\delta + n + g)k$$

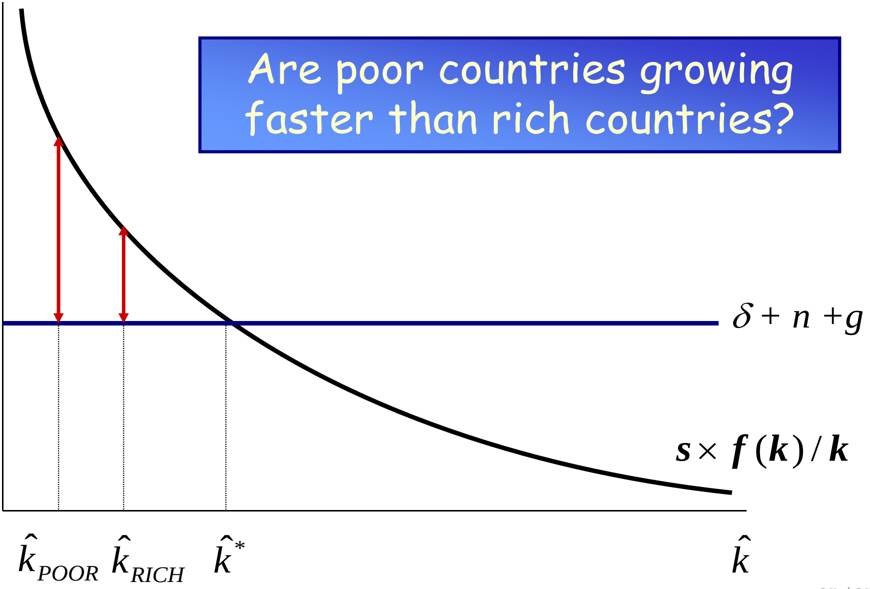
$$\frac{\Delta k}{k} = s\frac{f(k)}{k} - (\delta + n + g)$$

Growth rate of k

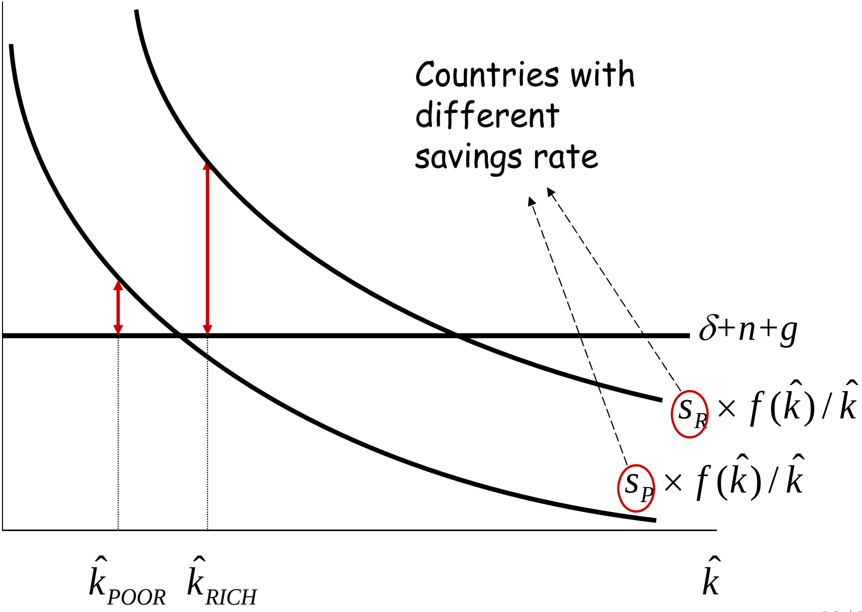
Changes over time

Constant

Conditional convergence



Why absolute convergence fails



Growth Accounting

Assume production function

$$Y = K^{\alpha}(EL)^{1-\alpha} = \underbrace{E^{1-\alpha}}_{=A} K^{\alpha} L^{1-\alpha},$$

where A is the the total factor productivity (TFP) then

$$\frac{\Delta Y}{Y} = \alpha \frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} + \frac{\Delta A}{A}$$

$\frac{\Delta A}{A}$ is also called the **Solow residual**—the contribution of TFP to output growth, not explainable by the growth in measurable factors of production (a “measure of our ignorance”).

Growth Accounting

- Solow (1957): developed the growth accounting framework and applied to U.S. data for assessment of the sources of growth during the early 20th century.
- Conclusion: a large part of of the growth was due to technological progress (growth in TFP)!

Source of Growth	Components	Data	Share of Output Growth
Output growth	$\Delta Y/Y$	3%	
Labour's share	$(1 - \alpha)$ times	0.67	
Labour growth rate	$\Delta L/L$	1%	
Contribution of labour			0.67
Capital's share	α times	0.33	
Capital growth rate	$\Delta K/K$	3%	
Contribution of capital			0.99
Contribution of productivity growth	$\Delta Y/Y - (1 - \alpha)(\Delta L/L) - \alpha(\Delta K/K)$		1.34
Proportion of growth due to increase in total factor productivity = 1.34/3 = 0.44			

Source: Authors' calculations.

Table 8-3 Accounting for Economic Growth in Canada
Mankiw and Scarth: Macroeconomics, Canadian Fifth Edition

Endogenous Growth Theory

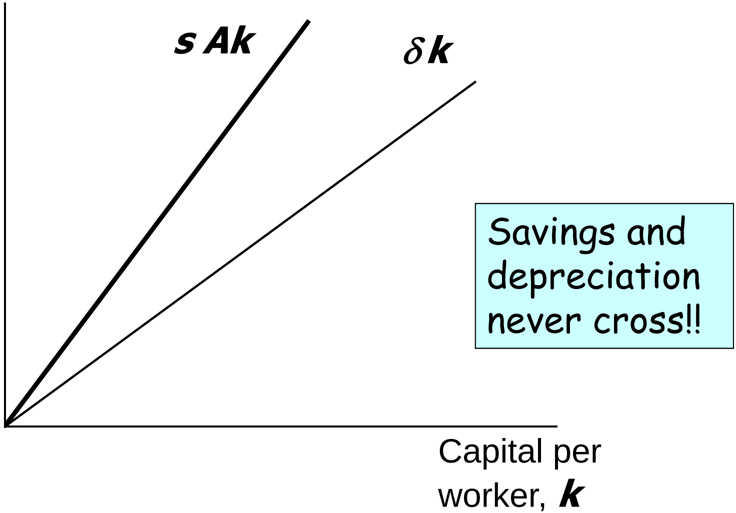
- Solow model:
  –sustained growth in living standards is due to tech progress
  –the rate of tech progress is **exogenous**
- Endogenous growth theory:
  –a set of models in which the growth rate of productivity and living standards is **endogenous**

Endogenous Growth models—AK model

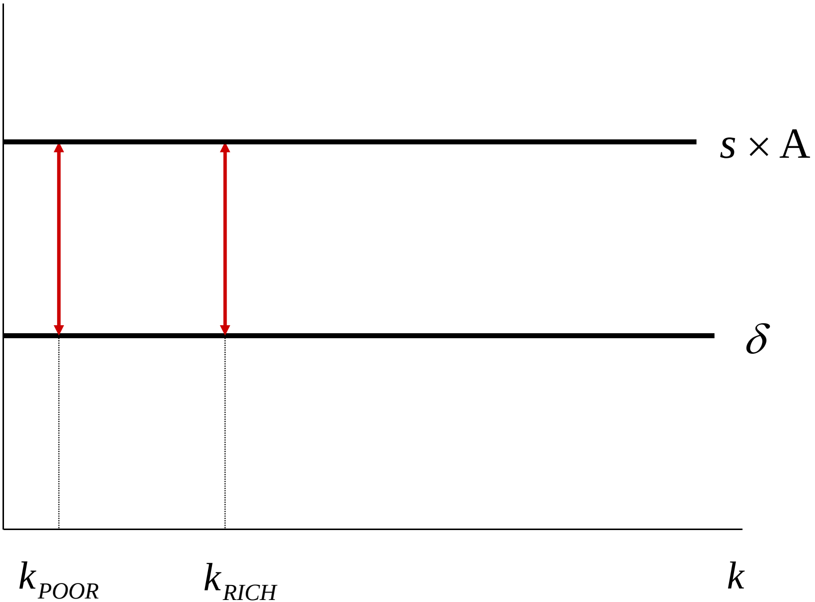
- Assume $Y = AK$ and A is some constant, and labor is not growing. Then $\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = \frac{sY - \delta K}{K} = sA - \delta$.
- Output per worker will grow forever if $sA > \delta$, and investment will be the engine of growth, since the growth rate will depend on s
- Policy changes (e.g., a change in savings) will have permanent **growth** effects
- $\Delta k = sAk - \delta k$

No SS in AK model

$$\Delta k = s\,Ak - \delta\,k$$



No convergence in AK model



- Important insight of AK models—sustained growth in output can be generated by the economy's fundamentals (A and s).
- Important feature of the production function that generates sustained growth—the returns to capital are constant, **not** diminishing. But...is it a reasonable assumption?

- No, if “capital” is narrowly defined (plants and equipment)
- Maybe yes with with a broad definition of “capital” (physical and human capital, knowledge)